A Treatise on the Holding Pattern:

Expelling the Myths and Misconceptions of Timing and Wind Correction

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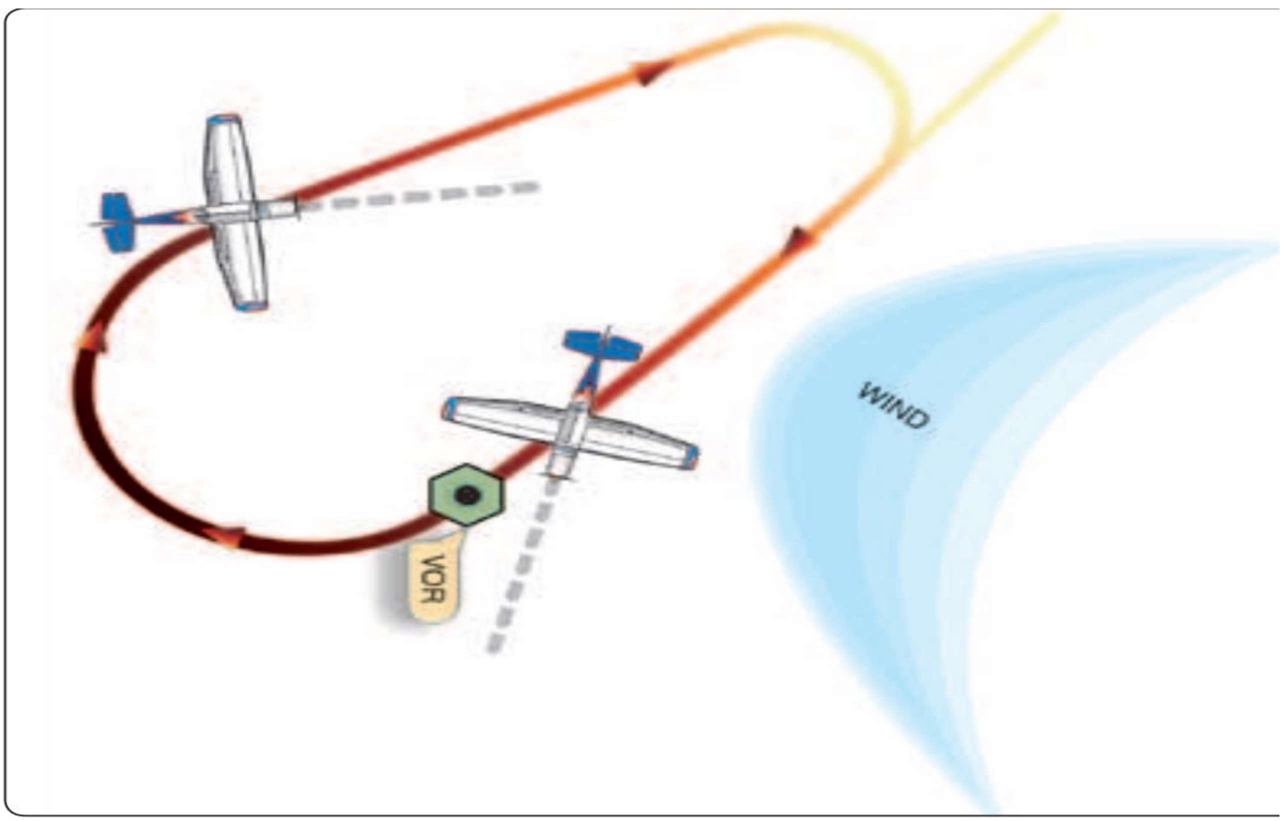

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Summary

As part of the ACS requirements for an Instrument rating, the Pilot must demonstrate an understanding and the required proficiency to fly a holding pattern. There are many training methods to provide the Pilot with the knowledge of how to visualize the holding pattern and enter the holding pattern. One of the required skills, is IR.III.B.S5, which states, "Uses proper wind correction procedures to maintain the desired holding pattern, and to arrive at the holding fix as close as possible to a specified time". The AIM provides some guidelines for estimating the outbound wind correction angle (OWCA), but there are no guidelines as under what conditions this rule-of-thumb should apply. In addition, there are no guidelines in the AIM for estimating the outbound time other than to fly a one-minute or one-minute and 30 second outbound leg for the initial circuit. The technique utilized to converge to the "Holding Pattern" solution is based on a bracketing technique, which in reality is a "Trial and Error" method. Here we fly a specified outbound OWCA and outbound time and based on the inbound time and whether the aircraft has undershot/overshot the centerline of the inbound course, the Pilot will fly the next circuit with an updated outbound time and OWCA. The process continues until the Pilot converges to the correct "Holding Pattern" solution. Depending on the initial guess for the outbound time and OWCA, the Pilot may require a significant number of circuits before converging to the correct holding pattern. This process of converging to the proper holding pattern can impose a considerable load on the Pilot, especially when attempting to troubleshoot a problem, or while reviewing the approach plate prior to executing the approach.

In order to expel many of the "Myths and Misconceptions" of timing and wind correction in the holding pattern, we derive the exact solution of the "Holding Pattern" problem. This solution is completely analytic and does not use graphical techniques to solve the problem, as utilized in many of the holding pattern calculators previously developed. It provides the following information to the IFR Pilot: (a) Inbound wind correction angle (IWCA), (b) Outbound heading or outbound wind correction angle (OWCA), and (c) outbound time. The solution of the "Holding Pattern" problem is shown to be a function of the following parameters:

(a) Windspeed ratio, $\bar{V}_W$, i.e. the ratio of the windspeed to the aircraft TAS (V_{TAS})
(b) Wind angle, α (degrees) relative to the inbound course to the holding fix
(c) Aircraft rate of turn, ω (radians/sec)
(d) Required inbound time to the fix (i.e. one-minute or one-minute and 30 seconds)

Note, although the shape of the holding pattern is a function of the above four parameters, the extent of the holding pattern (i.e. what the Radar Controller observes

on the radar scope) is also a function of the parameter $\frac{V_{TAS}}{\omega}$, since the $x-y$ coordinates of the holding pattern are proportional to this parameter.

The exact solution of the “Holding Pattern” problem allows the Pilot to not only have a better understanding of how to correct both the outbound heading and outbound time, but to be able to converge to the “Holding Pattern” solution in a minimum number of circuits. In addition, the exact solution provides a number of important properties about the holding pattern that have never previously been discussed in the open literature. This information can affect the way we train IFR Pilots in the future. Below are some of the key findings of this Treatise.

(1) There are two important advantages of starting the outbound time when the aircraft has turned to the outbound heading, rather than the abeam point. The first is obvious in that the Pilot does not need to locate the abeam point, and the second is that the outbound time measured from the time the aircraft reaches the outbound heading will be the same, regardless of whether the wind is blowing from either $\pm\alpha$, i.e. from the holding side or the non-holding side. If the Pilot starts the time at the abeam point, the outbound time will be different, depending on whether the wind is coming from the holding or non-holding side.

(2) A completely different type of holding pattern occurs when holding on a strong headwind component. In this type of holding pattern, it is impossible to achieve the one-minute or one-minute and 30 second inbound time unless the aircraft turns less than 90 degrees outbound from the inbound course. We define this holding pattern as a Type-2 holding pattern, as compared to the normal Type-1 holding pattern that we observe in IFR training manuals. We have derived the boundary of this type of holding pattern in windspeed-wind angle space (i.e. $\bar{V}_W-\alpha$ space). The boundary line is shown to be a function of the turn rate and the required inbound time to the holding fix. In the case of the one-minute inbound time, the Type-2 holding pattern will occur whenever the windspeed ratio becomes greater than $\frac{1}{3}$ while holding on a direct headwind. The value of $\bar{V}_W$ increase to 0.38 at $\alpha=45$, and 0.44 at $\alpha=60$ degrees. The behavior is similar for the one-minute and 30 second inbound time, except at $\alpha=0$, the value of $\bar{V}_W=\frac{3}{7}$ and increases with α in a similar fashion as the one-minute inbound leg case. We show that the Type-2 holding pattern can be extremely difficult to converge to the correct inbound time due to the required outbound turn being less than 90 degrees to the

inbound course. In fact, when the outbound turn is between 45 and 90 degrees from the inbound course, the inbound the time is controlled by the outbound heading, whereas, the overshoot/undershoot of the inbound course is controlled by the outbound time. This is exactly opposite to the “Bracketing Method” used for Type-1 holding patterns. Thus, by flying the holding pattern with a windspeed ratio less than $\frac{1}{3}$, the IFR Pilot can always avoid having to hold with a Type-2 holding pattern. In fact, it is recommended to fly the holding pattern with a value of $\bar{V}_W \le 0.25$, in order to have a sufficient amount of outbound time before having to turn to re-intercept the inbound course.

(3) The “Coupling Effect”: The concept of the “Coupling Effect”, which states that “Every Pilot induced change in the outbound time or OWCA causes changes in both the inbound time and the undershooting/overshooting of the inbound course to the fix”. This concept is extremely important in converging to the correct “Holding Pattern” solution using a minimal number of circuits. Using the exact solution of the “Holding Pattern” problem, we developed a “Smart-Convergence” algorithm, to converge to the correct holding pattern in a minimum number of circuits. This algorithm is compared to the current “Bracketing Method” and shows there are significant deficiencies in the “Bracketing Method” that requires additional circuits to converge to the correct holding pattern.

(4) We have developed curves of the exact solution for the standard Type-1 holding patterns for windspeed ratios up to 0.3, which show the outbound time and the ratio of the OWCA to the IWCA (i.e. the M-Factor) as a function of windspeed ratio and relative wind angle. These solutions show that using the AIM recommended M-Factor of 3 for the OWCA holds under a limited set of conditions. These conditions are: (a) For windspeed ratios up to 0.3, the relative wind angle is limited to the range $70 \le \alpha \le 95$ degrees, and (b) For $0 \le \alpha \le 180$ degrees, when the windspeed ratio is less than 0.05. For aircraft holding at a TAS of 100 knots, this would correspond to a wind of less than 5 knots. We identify this as one of the root causes of requiring additional circuits to converge to the correct holding pattern, since the initial circuit can be considerably different than the “Holding Pattern” solution. We have also shown that the bound on the M-Factor is given by

$$\frac{(3-\bar{V}_W)}{(1-3\bar{V}_W)} \ge \frac{OWCA}{IWCA} \ge \frac{(3+\bar{V}_W)}{(1+3\bar{V}_W)}$$

which debunks many of the articles in the open literature which claim that the M-Factor is between 2 and 3.

(5) We have developed some important techniques that can be used when flying Type-1 holding patterns, in order to converge to the “Holding Pattern” solution with a minimal number of circuits. These techniques are shown using actual tracks of the holding pattern while attempting to converge to the “Holding Pattern” solution. These curves are extremely helpful to the CFI-I when using the Simulator to introduce the IFR Student to holding patterns in the presence of a wind. In addition, just eyeballing the outbound time on this chart was shown to reduce the number of required circuits by 40 percent in order to converge to the correct holding pattern.

(6) We also discuss important IFR training methods that will improve the Student’s technique and understanding of wind correction and timing in the holding pattern. These techniques expel many of the “Myths and Misconceptions” of timing and wind correction in the holding pattern.

(7) This simple analysis for determining the outbound time and outbound heading should allow GPS manufacturers to implement a holding pattern page which contains all the information to properly fly the holding pattern. Since the winds can have variability over a period of 5-10 minutes, the GPS will have an update each time the aircraft reaches the holding fix and will provide the IFR Pilot with the outbound time and OWCA for the next circuit. With this GPS capability available, the IFR Pilot load during holding can be considerably reduced.

Finally, important formulas that are useful to both IFR Pilots and CFI-I’s are highlighted in red.

1.0 Introduction

As part of the training requirements for the Airplane Instrument Pilot rating, the Candidate must be proficient in the use of holding procedures. Holding patterns can be necessary for a number of reasons: (a) Delays at the airport of intended landing, (b) Loss of ATC communication, (c) Not prepared to execute the approach due to either equipment malfunction or under Single Pilot Operation, the Pilot may not be ready to execute the approach. However, whatever the need for the hold, the IFR Pilot should use this time in the holding pattern to prepare the aircraft for the approach.

The latest ACS for the Airplane Instrument Pilot rating requires both knowledge and skills in mastering the hold while flying in the presence of a wind. In particular, IR.III.B.S5 states "Uses proper wind correction procedures to maintain the desired pattern and to arrive over the fix as close as possible to a specified time and maintain pattern leg lengths when specified". The AIM (Par 5-3-8) provides a number of guidelines and rules-of-thumb for flying the hold in the presence of a wind. For example, in terms of the outbound heading, the AIM recommends determining the inbound wind correction angle (IWCA) and multiplying it by 3 (i.e. the M-Factor) to determine the outbound wind correction angle (OWCA). In regard to the outbound time, the AIM recommends on the first circuit, using one minute (or one minute and 30 seconds) for the outbound time measured from the abeam point of the holding fix. If the abeam point cannot be determined, then use the outbound heading as the point to initiate the outbound time. After the first circuit, correct the outbound time to achieve the specified inbound time. Note that this process of converging to the holding pattern is based on a "Bracketing Method" or "Trial and Error Method". Although the AIM does not recommend any rules-of-thumb for correcting the outbound time for the next circuit, there have been numerous rules-of-thumb proposed in IFR training manuals such as Ref. 1. However, these rules-of-thumb do not come with any specific limitations.

In order to overcome the problem of converging to the correct holding pattern, Holding Pattern Calculators were developed in an attempt to provide the IFR Pilot with both the outbound heading and outbound time, given the windspeed and direction. These calculators were very complex and used graphical methods to generate the outbound time and heading. In addition, as the windspeed increased beyond approximately 0.25, these calculators were found to be inaccurate.

In order to reduce the number of circuits that the IFR Pilot needs to converge to the correct holding pattern, as well as expel many of the "Myths and Misconceptions" of timing and wind correction in the holding pattern, we derive the exact solution of the "Holding Pattern" problem. To the Author's knowledge, the exact solution of the "Holding Pattern" problem has never been documented in the open literature. This solution is both analytic and exact and thus does not contain any limitations in terms of the windspeed or direction. The work in this Treatise is based on an earlier FAASTeam Seminar on holding patterns, presented in June 2013 in Van Nuys California (Ref. 2).

In Section 2 we derive the exact solution of the ”Holding Pattern” problem. The extent of the Mathematics required involves both elementary Trigonometry and Algebra, plus a small amount of elementary Calculus. Using the exact solution, we discover a number of interesting properties of the holding pattern including a completely different type of holding pattern that arises under a strong wind with a headwind component on the inbound course to the fix. We define this new pattern as a Type-2 holding pattern, compared to the standard Type-1 holding pattern that is documented in many of the IFR training manuals. In addition, in the case of Type-1 holding patterns, we develop simple curves for the M-Factor, OWCA and outbound time as a function of the relative wind angle, for windspeed ratios up to 0.3. We also develop the boundary line between Type-1 and Type-2 holding patterns in windspeed ratio-wind angle space.

In Section 3, we utilize the exact solution to develop a “Smart-Convergence” algorithm that allows the Pilot to converge to the “Holding Pattern” solution in a minimum number of circuits. Here we introduce the concept of the “Coupling Effect” and show that one of the root causes of requiring a large number of circuits to converge to the “Holding Pattern” solution, is due to a lack of understanding of the importance of including the “Coupling Effect” in the convergence process.

In Section 4, we compare the “Smart-Convergence” algorithm with the “Bracketing Method” and show how the “Bracketing Method” is inefficient in converging to the correct holding pattern. In Section 5 we discuss how to prepare for the hold, and some simple techniques to use which can reduce the number of circuits to converge to the holding pattern while using the “Bracketing Method”. In Section 6 we develop training techniques that should be included when discussing timing and wind correction in the holding pattern. In Section 7 we summarize the conclusions drawn from the work in this Treatise, and in Section 8 we list the references. Finally, Appendix A contains all the elementary mathematics written as a script, for those readers having access to Matlab software. Running this script in Matlab will provide the track of the aircraft in the holding pattern while converging to the ”Holding Pattern” solution”.

2.0 Exact Solution of the "Holding Pattern" Problem

The exact solution of the "Holding Pattern" problem provides the following information to the pilot:

(1) Inbound wind correction angle (IWCA)
(2) Outbound heading and outbound wind correction angle (OWCA)
(3) Outbound time measured from the point at which the aircraft has completed its turn to the outbound heading

However, it is important to understand both the key parameters that affect the shape and extent of the holding pattern, and the actual dimensions of the holding pattern. As in all ground reference maneuvers, there are two key parameters that come into play when tracking the inbound course in the holding pattern. These are: (a) The windspeed ratio $\bar{V}_W = \dfrac{V_W}{V_{TAS}}$, and (b) The angle α, which is the relative angle between the wind direction and the inbound course to the holding fix. In Ref. 3, we derived the solution of the "Wind Triangle" problem, which provides both the groundspeed and the wind correction angle (WCA) σ. The WCA while tracking a particular course was found to be

$$\mathrm{Sin}\,\sigma = \bar{V}_W\,\mathrm{Sin}\,\alpha \qquad (1)$$

Thus, once the windspeed ratio and relative wind angle α are defined, the WCA is automatically determined from eq. (1). The non-dimensional groundspeed along the particular course to be tracked is given by

$$\bar{V}_G = \frac{V_G}{V_{TAS}} = \mathrm{Cos}\,\sigma - \bar{V}_W\,\mathrm{Cos}\,\alpha \qquad (2)$$

There are two additional parameters that characterize the holding pattern. These are: (a) Outbound heading (θ_H), and (b) Outbound time (t_{out}). Both parameters depend on: (1) Windspeed ratio ($\bar{V}_W$), (2) Relative wind angle (α), (3) Aircraft turn rate (ω), and (4) Required inbound time to the holding fix.

In order to characterize the holding pattern, we define a simple x-y Cartesian Coordinate system, where we locate the holding fix at the point x=0, y=0 and the inbound course along the negative-x axis. In Figure 1, we show a non-standard holding pattern, i.e. left-hand turns. We define point 1 as the location of the aircraft at the point the aircraft has turned to the outbound heading (θ_H). We define point 2 as the location of the aircraft at the end of the outbound time (t_{out}) at the point where it begins its turn to intercept the inbound course, and point 3, the location of the aircraft at the point at which the aircraft has re-intercepted the inbound course with the appropriate inbound WCA (IWCA).

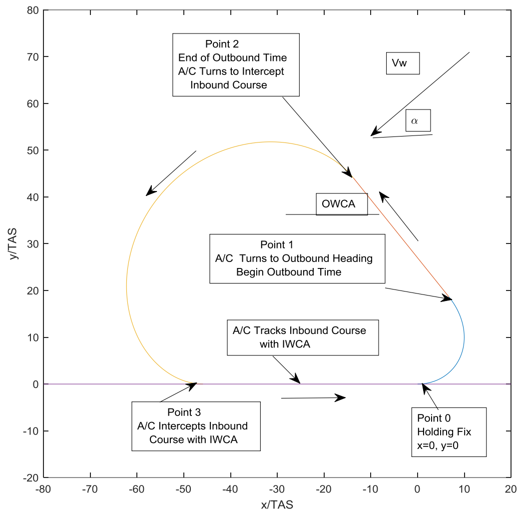


Figure 1: Non-Standard Holding Pattern

Since the IWCA is known, the only remaining unknowns are the outbound heading, θ_H ,and the outbound time, t_{out}. The determination of these two unknowns will require two equations to solve for these unknowns. Note that in Figure 1, we see that when the aircraft reaches the holding fix, it initiates a turn and tracks an arc from point 0 to point 1, where the aircraft has turned to the outbound heading θ_H . The aircraft then flies at a constant heading between the points 1 and 2 for a period of time given by t_{out}. The aircraft then initiates a turn at point 2 and at point 3, rolls out on a heading which includes the inbound course plus the IWCA. The aircraft then tracks the inbound course for the prescribed amount time (i.e. one-minute below 14000MSL, or one-minute and 30 seconds at or above 14000MSL, as per the AIM 5-3-8).

Since the aircraft initiated the arc at point 0 (i.e. x=0, y=0), it must return to point 3 at the same value of y, i.e. $y=0$. Thus, if one calculates the changes in the value of y in going from segments 0-1, 1-2 and 2-3, these changes must sum to zero. This is the first constraint necessary to obtain the "Holding Pattern" solution. The second equation is obtained by calculating the changes in the value of x along the 3 segments 0-1, 1-2, and 2-3 and then adding the groundspeed along the segment 3-0 multiplied by the prescribed inbound time (i.e. one-minute or one-minute and 30 seconds) and require the sum of all the changes in x to be identically zero (i.e. aircraft ends up at x=0). Using these two equations, one can determine both the unknown outbound heading θ_H , and outbound time t_{out}.

Without any loss in generality, we can assume the inbound course is 0 degrees. In this x-y Cartesian Coordinate system, the angle θ , represents the aircraft heading relative to the inbound course, where θ is measured from the positive-x axis in the counterclockwise direction. This relative heading should not be confused with the heading observed on the heading indicator. One can easily overlay the heading indicator onto Figure 1 and obtain the actual outbound heading. However, it is much simpler to work with relative heading.

In order to determine the actual track of the aircraft in the holding pattern, it is necessary to determine the aircraft groundspeed in the x-y Cartesian Coordinate system, i.e. the components of the groundspeed in the x and y directions. The components of the groundspeed in both x and y directions are given by

$$\begin{aligned} V_{G_x} &= V_{TAS}\,\mathrm{Cos}\,\theta - V_W\,\mathrm{Cos}\,\alpha \\ V_{G_y} &= V_{TAS}\,\mathrm{Sin}\,\theta - V_W\,\mathrm{Sin}\,\alpha \end{aligned} \qquad (3)$$

When the aircraft is on a constant heading (i.e. constant θ), the groundspeed will be constant. However, when the aircraft is turning, the groundspeed will be varying. Note that when the aircraft is at constant groundspeed, the change in the x-y coordinates of the aircraft are just given by the groundspeed multiplied by the time of flight. However, when the groundspeed is varying, one must compute the change in position of the aircraft by performing an integration of the varying groundspeed multiplied by an element of time, and then integrated over a time interval $t_f - t_i$. Here t_i is the time at the beginning of the turn, and t_f is the time at the end of the turn, i.e.

$$\begin{aligned} \Delta x &= \int_{t_i}^{t_f} V_{G_x}\,dt \\ \Delta y &= \int_{t_i}^{t_f} V_{G_y}\,dt \end{aligned} \qquad (4)$$

During the turning portion of the holding pattern, the aircraft rate of turn in radians/sec is given by

$$\omega = \frac{g\,\mathrm{Tan}\,\phi}{V_{TAS}} \qquad (5)$$

Here, g is the gravitational acceleration (i.e. 32.174 $\frac{ft}{\sec^2}$), V_{TAS} the TAS in ft/sec, and ϕ is the aircraft bank angle in degrees. In order to convert the TAS in knots to ft/sec, we need to multiply the TAS by 1.6875. In general, the aircraft will be turning at a standard rate of 3 degrees/sec, up to the point where the bank angle reaches 30 degrees (or 25 degrees while using a flight director). Using eq. (5), one can see this will occur at 210 knots for a 30-degree bank, and 170 knots when using a flight director.

In order to perform the integration shown in eq. (4), it is best to transform the element of time, dt, into an element of heading change, $d\theta$. Since the turn rate is constant during the turning portion of the flight, we can express dt in terms of dθ, i.e.

$$d\theta = \omega dt \qquad (6)$$

One can now rewrite eq. (4) as

$$\Delta x = \frac{1}{\omega}\int_{\theta i}^{\theta_f} V_{G_x} d\theta$$
$$\Delta y = \frac{1}{\omega}\int_{\theta_i}^{\theta_f} V_{G_y} d\theta \qquad (7)$$

Where θ_i is the aircraft heading at the beginning of the turn, and θ_f is the aircraft heading at the completion of the turn. If we normalize Δx and Δy by $\frac{V_{TAS}}{\omega}$, eq. (7) becomes

$$\Delta\bar{x} = \frac{\omega\Delta x}{V_{TAS}} = \int_{\theta_i}^{\theta_f} \bar{V}_{G_x} d\theta$$
$$\Delta\bar{y} = \frac{\omega\Delta y}{V_{TAS}} = \int_{\theta_i}^{\theta_f} \bar{V}_{G_y} d\theta \qquad (8)$$

Where the normalized groundspeed is given by

$$\bar{V}_{G_x} = \mathrm{Cos}\,\theta - \bar{V}_W\,\mathrm{Cos}\,\alpha$$
$$\bar{V}_{G_y} = \mathrm{Sin}\,\theta - \bar{V}_W\,\mathrm{Sin}\,\alpha \qquad (9)$$

and $\frac{V_{TAS}}{\omega}$ is the radius of the turn under no-wind conditions. Note that the shape of the holding pattern when expressed in normalized coordinates is a function of the windspeed ratio, $\bar{V}_W$, the angle of the wind relative to the inbound course, α, the outbound heading, θ_H, and the outbound time, t_{out}. However, the actual extent of the holding pattern (i.e. what the Radar Controller will see on his screen), will depend on the value of $\frac{V_{TAS}}{\omega}$, since every normalized value of $\bar{x}$ and $\bar{y}$ will be multiplied by this quantity.

Equations. (8) and (9) can now be utilized to obtain the two equations necessary to determine the outbound time and the outbound heading required to intercept the inbound course with either a one-minute, or one minute and 30 second inbound leg to the holding fix. If we substitute eq.(9) into eq. (8), we obtain the following equations for the normalized values of x and y during the turning portion of the holding pattern, i.e.

$$
\begin{aligned}
\Delta\bar{x} &= (\text{Sin}\,\theta_f - \text{Sin}\,\theta_i) - \bar{V}_W\,\text{Cos}\,\alpha(\theta_f - \theta_i) \\
\Delta\bar{y} &= -[(\text{Cos}\,\theta_f - \text{Cos}\,\theta_i) + \bar{V}_W\,\text{Sin}\,\alpha(\theta_f - \theta_i)]
\end{aligned}
\qquad (10)
$$

Equation (10) can be utilized for the turning segments, i.e., segments 0-1, and 2-3.

The changes in $\Delta\bar{x}$ and $\Delta\bar{y}$ along the straight segments 1-2 and 3-0, where the groundspeed is constant, are given by

$$
\begin{aligned}
\Delta\bar{x} &= (\text{Cos}\,\theta - \bar{V}_W\,\text{Cos}\,\alpha)\omega t \\
\Delta\bar{y} &= (\text{Sin}\,\theta - \bar{V}_W\,\text{Sin}\,\alpha)\omega t
\end{aligned}
\qquad (11)
$$

Where t represents either the unknown outbound time from point 1 to 2, or the known inbound time from point 3 to 0. In regard to the aircraft headings, $\theta = \theta_H$ is the unknown outbound heading from point 1 to 2, and $\theta = 2\pi + \sigma$ is the aircraft heading after completing a 360-degree turn. Here, σ is the IWCA while tracking the segment from points 3 to 0.

We should point out that we have assumed that at the appropriate times, the turn rate instantaneously changes from either zero to the value ω, or from the value ω to zero. If the rate of roll-in and roll-out is similar, one would expect this assumption to have a minor effect on the accuracy of the solution.

We will start by calculating the changes in $\Delta\bar{y}$ corresponding to the 3 segments, 0-1, 1-2, and 2-3. The changes in $\Delta\bar{y}$ are given by

$$
\begin{aligned}
\Delta\bar{y}_{0-1} &= -(\text{Cos}\,\theta_H - \text{Cos}\,\sigma) - \bar{V}_W\,\text{Sin}\,\alpha(\theta_H - \sigma) \\
\Delta\bar{y}_{1-2} &= (\text{Sin}\,\theta_H - \bar{V}_W\,\text{Sin}\,\alpha)\omega t_{out} \\
\Delta\bar{y}_{2-3} &= -(\text{Cos}(2\pi + \sigma) - \text{Cos}\,\theta_H) - \bar{V}_W\,\text{Sin}\,\alpha[(2\pi + \sigma) - \theta_H]
\end{aligned}
\qquad (12)
$$

Note that although the aircraft heading comes back to its original heading on the inbound leg, it has turned 360 degrees (i.e. 2π radians). In the presence of a wind, the aircraft total time is key factor, and thus the 360 degrees must be taken into account.

Since the sum of the $\Delta\bar{y}$'s need to be identically zero in order for the aircraft to re-intercept the inbound course at the time the aircraft has turned to the inbound course plus the inbound WCA, we can set the sum equal to zero and solve for the unknown outbound time, t_{out}, i.e.,

$$
t_{out} = \frac{(\frac{2\pi}{\omega})}{[(\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma}) - 1]} \qquad (13)
$$

Where ω is the turn rate in radians/sec. We can convert degrees/sec to radians/sec using the following formula

$$\omega = \frac{\pi}{180} k \qquad (14)$$

Where k is the aircraft turn rate in degrees/sec. Substituting eq. (14) into eq. (13), gives the following equation for the outbound time in seconds between point 1 and 2, i.e.

$$t_{out} = \frac{(\frac{360}{k})}{[(\frac{\operatorname{Sin}\theta_H}{\operatorname{Sin}\sigma}) - 1]} \qquad (15)$$

Note that the outbound time is a function of the turn rate k, the outbound heading θ_H , and the IWCA σ . Since we know the turn rate and the IWCA, the only unknown is the outbound heading θ_H .In the case of an aircraft performing a standard rate turn, the outbound time will be given by

$$t_{out} = \frac{120}{[(\frac{\operatorname{Sin}\theta_H}{\operatorname{Sin}\sigma}) - 1]} \qquad (16)$$

We now develop the equation to determine the outbound heading θ_H . We will start by calculating the changes in the $\Delta\bar{x}$'s corresponding to the 3 segments, 0-1, 1-2, and 2-3. The changes in $\Delta\bar{x}$ are given by

$$\begin{aligned} \Delta\bar{x}_{0-1} &= (\operatorname{Sin}\theta_H - \operatorname{Sin}\sigma) - \bar{V}_W \operatorname{Cos}\alpha(\theta_H - \sigma) \\ \Delta\bar{x}_{1-2} &= (\operatorname{Cos}\theta_H - \bar{V}_W \operatorname{Cos}\alpha)\omega t_{out} \\ \Delta\bar{x}_{2-3} &= [\operatorname{Sin}(2\pi + \sigma) - \operatorname{Sin}\theta_H] - \bar{V}_W \operatorname{Cos}\alpha[(2\pi + \sigma) - \theta_H] \end{aligned} \qquad (17)$$

If we add the 3 values of Δx together, it will place the aircraft at point 3. The distance from point 3 to point 0 must be equal to

$$\Delta\bar{x}_{3-0} = [\operatorname{Cos}(2\pi + \sigma) - \bar{V}_W \operatorname{Cos}\alpha]\omega t_{in} \qquad (18)$$

Thus, the equation that determines θ_H is given by

$$\Delta\bar{x}_{0-1} + \Delta\bar{x}_{1-2} + \Delta\bar{x}_{2-3} + \Delta\bar{x}_{3-0} = 0 \qquad (19)$$

Note that $\operatorname{Sin}(2\pi + \sigma) = \operatorname{Sin}\sigma$, and $\operatorname{Cos}(2\pi + \sigma) = \operatorname{Cos}\sigma$. If we substitute eqs. (17) and eq. (18) into eq. (19) we obtain the following equation for θ_H

$$-2\pi\bar{V}_W \operatorname{Cos}\alpha + (\operatorname{Cos}\theta_H - \bar{V}_W \operatorname{Cos}\alpha)\omega t_{out} + [\operatorname{Cos}(2\pi + \sigma) - \bar{V}_W \operatorname{Cos}\alpha]\omega t_{in} = 0 \qquad (20)$$

The required inbound time t_{in}, is given by

$$t_{in} = 60\beta \qquad (21)$$

Where β is 1 for a one-minute inbound leg, and $\frac{3}{2}$ for a one-minute and 30 second inbound leg. If we now substitute eq. (15) for t_{out}, and eq. (21) for t_{in}. we obtain the following equation for θ_H

$$a_1 \operatorname{Sin}\theta_H + a_2 \operatorname{Cos}\theta_H + a_3 = 0 \qquad (22)$$

Where

$$a_1 = \operatorname{Cos}\sigma - (1 + \frac{6}{k\beta})\bar{V}_W \operatorname{Cos}\alpha$$
$$a_2 = \frac{6}{k\beta}\operatorname{Sin}\sigma \qquad (23)$$
$$a_3 = -(\operatorname{Cos}\sigma - \bar{V}_W \operatorname{Cos}\alpha)\operatorname{Sin}\sigma$$

Note that although the equation for the outbound time t_{out} is exact and in analytic form, eq. (22) is a transcendental equation for θ_H , and thus must be solved by numerical root-finding methods. However, we should point out that eq. (22) is also an exact solution for θ_H . Equation (23) shows that the for a given windspeed ratio and relative wind angle, the outbound heading is a function of the product $k\beta$.

If one is interested in obtaining an analytical solution to eq. (22), we first replace $\operatorname{Sin}\theta_H$ with $\sqrt{1-\operatorname{Cos}^2\theta_H}$. By eliminating $\operatorname{Sin}\theta_H$ from eq. (22), we obtain the following quadratic equation for $\operatorname{Cos}\theta_H$

$$(a_1^{\,2} + a_2^{\,2})\operatorname{Cos}^2\theta_H + 2a_2 a_3 \operatorname{Cos}\theta_H + (a_3^{\,2} - a_1^{\,2}) = 0 \qquad (24)$$

Solving the above quadratic equation for $\operatorname{Cos}\theta_H$

$$\operatorname{Cos}\theta_H = \frac{[-a_2 a_3 \pm a_1\sqrt{a_1^{\,2} + a_2^{\,2} - a_3^{\,2}}\,]}{a_1^{\,2} + a_2^{\,2}} \qquad (25)$$

Note that eq. (25) contains two possible solutions as can be seen with the $\pm$ sign. We must choose the negative sign in order that the relative outbound heading lies in the range $0 \le \theta_H \le 180$ degrees. Thus, the final equation for the outbound heading is given by

$$\operatorname{Cos}\theta_H = \frac{-[a_2 a_3 + a_1\sqrt{a_1^{\,2} + a_2^{\,2} - a_3^{\,2}}\,]}{a_1^{\,2} + a_2^{\,2}} \qquad (26)$$

Taking the inverse Cosine of eq. (26), we get

$$\theta_H = \text{Cos}^{-1}[\frac{-(a_2 a_3 + a_1\sqrt{a_1^2 + a_2^2 - a_3^2})}{a_1^2 + a_2^2}] \qquad (27)$$

Since the outbound time t_{out} requires $\text{Sin}\,\theta_H$, we can obtain the required expression for $\text{Sin}\,\theta_H$ in one of two ways, i.e.

$$\text{Sin}\,\theta_H = \text{Sin}\{\text{Cos}^{-1}[\frac{-(a_2 a_3 + a_1\sqrt{a_1^2 + a_2^2 - a_3^2})}{a_1^2 + a_2^2}]\} \qquad (28)$$

or from the identity

$$\text{Sin}\,\theta_H = \sqrt{1 - \text{Cos}\,\theta_H^{\,2}} \qquad (29),$$

Where eq. (26) is utilized in eq. (29) to obtain $\text{Sin}\,\theta_H$ .Thus, the “Holding Pattern” solution for an arbitrary windspeed and direction is given by eqs.(1), (15), and (26)-(29).

An additional parameter that is useful to the IFR Pilot is the total time for one circuit of the holding pattern. It is easily to obtain this parameter since it is given by

$$tth_c = \frac{360}{k} + t_{out} + 60\beta \qquad (30)$$

Where the first term is the time to perform a 360-degree turn, the second term is the outbound time, and the third term is just the required inbound time.

Substituting eq. (15) into eq. (30), we obtain the following equation for the total time in seconds for one circuit in the holding pattern

$$tth_c = \frac{60\beta[(1+\frac{6}{k\beta})\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} - 1]}{[\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} - 1]} \qquad (31)$$

In the case of the aircraft performing a standard rate turn (k=3), and a one-minute inbound leg ($\beta = 1$), eq. (31) becomes

$$tth_c = 60\frac{[3\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} - 1]}{[\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} - 1]} \qquad (32)$$

Thus, the total time in the holding pattern for a one-minute inbound time with a standard rate turn is a function of both the outbound heading θ_H , and the IWCA σ .

In order to understand how to obtain the "Holding Pattern" solution for any wind condition, we use the following method:

(1) Select the wind speed and the wind direction
(2) Select the V_{TAS}
(3) Determine the windspeed ratio $\bar{V}_W = \frac{V_{wind}}{V_{TAS}}$
(4) Determine the value of the wind direction (α) relative to the inbound course
(5) Select the aircraft desired rate of turn k in degrees/sec (i.e. either standard rate or bank angle limited)
(6) Select the inbound time (β=1 below 14000MSL and $\frac{3}{2}$ at and above 14000MSL)
(7) Calculate the IWCA σ from eq. (1)
(8) Calculate the a_1, a_2, and a_3 coefficients from eq. (23)
(9) Solve eqs. (26) and (27) for θ_H
(10) Solve eq. (29) for $\mathrm{Sin}\,\theta_H$, and eq. (15) for the outbound time t_{out}
(11) Solve eq. (31) for the total time for one circuit of the holding pattern

Although the above analysis was developed to satisfy a required time for the inbound leg, i.e. 60β, it can easily be extended to satisfying a defined length of the inbound leg. Equation (18) defines the length of the normalized inbound leg. We can solve for the value of β that meets the required length of the inbound leg L_{IC}, i.e.

$$\frac{L_{IC}}{V_{TAS}} = 60\beta[\mathrm{Cos}(2\pi+\sigma) - \bar{V}_W\,\mathrm{Cos}\,\alpha] \qquad (33)$$

Where L_{IC} is in nm, V_{TAS} is the true airspeed in nm/sec (i.e. divide the TAS in knots by 3600). Solving for the unknown value of β

$$\beta = \frac{L_{IC}}{60V_{TAS}[\mathrm{Cos}(2\pi+\sigma) - \bar{V}_W\,\mathrm{Cos}\,\alpha]} = \frac{L_{IC}}{60V_{TAS}[\mathrm{Cos}\,\sigma - \bar{V}_W\,\mathrm{Cos}\,\alpha]} \qquad (34)$$

The value of β determined by eq. (34) is substituted into the previous equations to determine the outbound heading and outbound time, which will allow the aircraft to re-intercept the inbound course with the required IWCA at a distance L_{IC} from the holding fix.

Although the above analysis was derived for a non-standard holding pattern (i.e. left turns), the same equations can be employed for the standard holding pattern (i.e. right turns), if the definition of the relative heading θ is positive and increasing in the clockwise direction. In both left and right turns, positive α is a wind coming from the holding side. Finally, in order to obtain the correct ground track for the standard holding pattern, the changes in $\Delta\bar{y}$ given in eq. (12) need to be multiplied by -1. Note, a change in the sign of $\Delta\bar{y}$ does not affect the outbound time equation since all the $\Delta\bar{y}$'s must sum to zero.

In Section 2.1 we will analyze the exact “Holding Pattern” solution in more detail in order to discover a number of interesting properties that have never been previously discussed in the open literature.

2.1 Properties of the Holding Pattern

Let us consider the case where the wind is coming from the relative direction $-\alpha$ instead of $+\alpha$. In this case the headwind component on the inbound leg is identical, however in the $-\alpha$ case, the wind is coming from the non-holding side rather than the holding side. In this case the WCA is $-\sigma$ rather than $+\sigma$.

Using the fact that

$$\begin{aligned} \text{Sin}(-\sigma) &= -\text{Sin}\,\sigma \\ \text{Cos}(-\sigma) &= \text{Cos}\,\sigma \end{aligned} \qquad (35)$$

It is easy to see that the solution for θ_H in the case $\alpha = -\alpha$ is just

$$\theta_H = -\theta_H \qquad (36)$$

Since the ratio

$$\frac{\text{Sin}(-\theta_H)}{\text{Sin}(-\sigma)} = \frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} \qquad (37)$$

we see that the outbound time t_{out} is the same regardless of whether the wind is coming from the holding side or the non-holding side. It is important to point out that although the outbound time from point 1 to 2 is the same in both cases, the outbound time measured from the abeam point of the holding fix will not be the same. This can be seen in Figure 2, for the case $\bar{V}_W = 0.3$ and $\alpha = \pm 45$ degrees. Note that $\alpha = -45$ is equivalent to $\alpha = 315$ degrees as shown in the Figure. In addition, the x-y coordinates are normalized by the TAS in nm/sec.

Thus, there are two distinct advantages of starting the outbound time at the point where the aircraft has turned to the outbound heading: (1) The outbound time is the same whether the wind direction is $\pm\alpha$, and (2) The abeam point does not have to be determined. Removing the requirement of starting the outbound time at the abeam point, reduces the required IFR Pilot workload, since the location of the abeam point is no longer necessary. Although the AIM states that the outbound time should be started at the abeam point, if it can be identified, the only requirement in the AIM is to meet the one-minute or one-minute and 30 second inbound time requirement. Thus, starting the outbound time when the aircraft reaches the outbound heading has considerable advantages and should be utilized while flying the holding pattern.

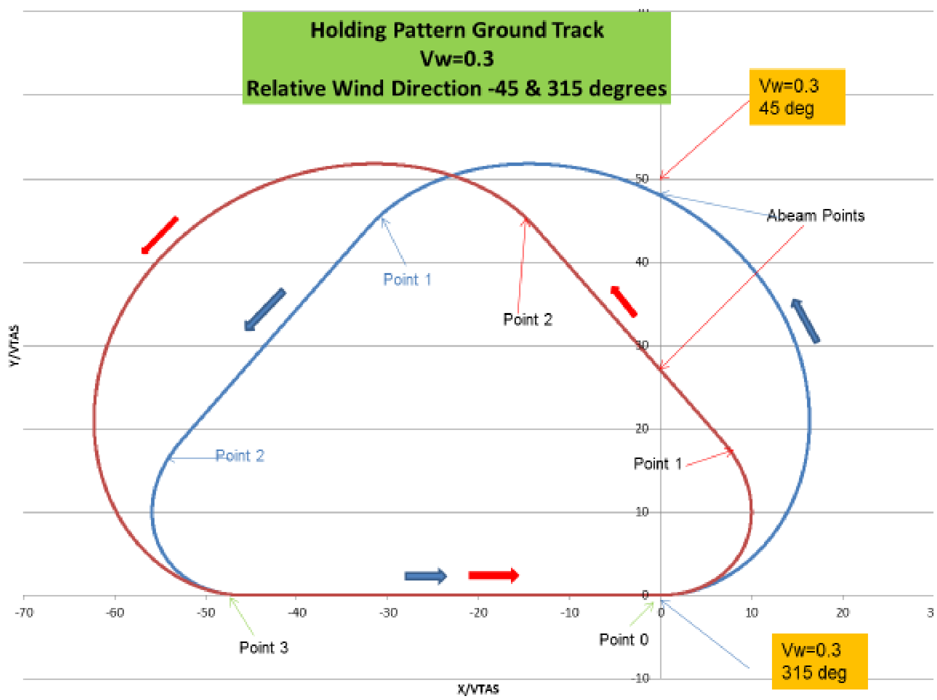


Figure 2: Comparison of Holding Patterns for the Cases: $\bar{V}_W = 0.3$, $\alpha = \pm 45$

There is another type of holding pattern that can exist when the IFR Pilot attempts to hold in the presence of a strong headwind. This type of holding pattern has never been previously discussed in the open literature. In this holding pattern, it is impossible for the Pilot to meet the required inbound time to the fix unless the aircraft reaches the fix and then turns to an outbound heading that is less than 90 degrees relative to the inbound course. We identify this holding pattern as a Type-2 pattern. We will refer to a holding pattern which requires a turn of more than 90 degrees from the inbound course as a Type-1 holding pattern. The Type-1 holding patterns are always shown in current IFR training manuals.

In order to determine under what conditions the Type-2 holding pattern can exist, we seek a solution of eq. (22) assuming the relative outbound heading $\theta_H = 90$ degrees. Since

$$\begin{aligned}\text{Sin}(90) &= 1 \\ \text{Cos}(90) &= 0\end{aligned} \qquad (38)$$

eq. (22) becomes

$$a_1 + a_3 = 0 \qquad (39)$$

Substituting eq. (23) into eq.(39) we obtain the following equation for $\bar{V}_W$

$$\sqrt{1-\bar{V}_W{}^2 \,\text{Sin}^2 \alpha}\,(1-\bar{V}_W \,\text{Sin}\,\alpha) - \bar{V}_W \sqrt{1-\text{Sin}^2 \alpha}\,[(1+\frac{6}{k\beta}) - \bar{V}_W \,\text{Sin}\,\alpha] = 0 \qquad (40)$$

We can solve this equation for $\bar{V}_W$ as a function of α. Figure 3a, corresponding to the case k=3 and b=1, shows a plot of $\bar{V}_W$ as a function of α, where α is in the range zero to ninety degrees (i.e. a headwind component on the inbound leg). We see that when we have a direct headwind on the inbound leg, windspeed ratios greater than 1/3 will require the pilot to track outbound on the 360-degree course from the fix (i.e. positive x-axis) for a specified time before turning back. In addition, we see that as the relative wind angle moves toward the aircraft’s wingtip, the Type-2 holding pattern will occur at higher windspeed ratios. Figure 3b shows a comparison of the boundary line for the two cases b=1 and b=3/2. We observe that the Type-2 holding pattern occurs at higher values of $\bar{V}_W$ when b=3/2 as compared to the b=1 case. However, flying the holding pattern at a windspeed ratio less than $\frac{1}{3}$ will avoid the Type-2 holding pattern for both values of b.

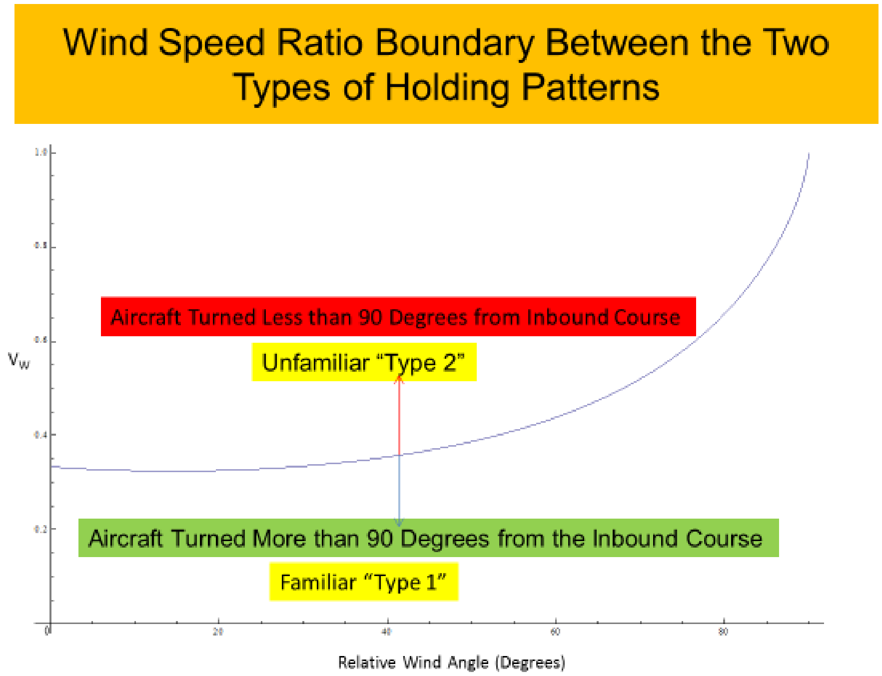


Figure 3a: Boundary Line between Type-1 and Type-2 Holding Pattern (k=3, b=1)

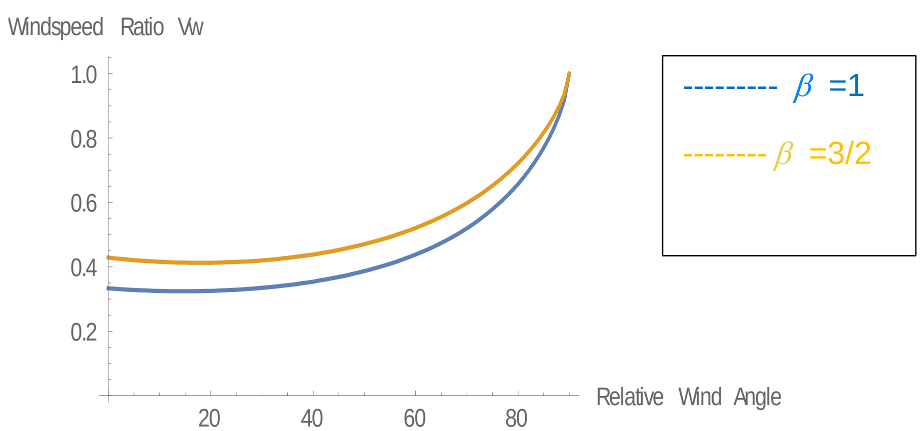


Figure 3b: Comparison of Boundary Line between Type-1 and Type-2 Holding Patterns (k=3, β =1 and β =3/2)

Another issue that arises for all IFR Pilots after entering the hold, is the aircraft initially flies outbound for one minute or one minute and 30 seconds and then turns back to re-intercept the inbound course to the holding fix. Let's assume that the aircraft intercepts the inbound course and reaches the holding fix in 40 seconds. The question that arises is: How much additional time should be added to the original outbound time in order that the next inbound time will be one minute? Some CFI-I's use the rule of thumb $\Delta t_{out} \simeq \Delta t_{in}$, however, we will show that this approximation can be in considerable error when the windspeed ratio is not very small.

In order to understand the outbound time correction issue, it is important to remember that all corrections to the outbound time are performed on the outbound leg where the aircraft heading is held constant. Thus, the additional distance parallel to the x-axis that is flown on the outbound leg due to a change in outbound time, Δt_{out} , will be equal the additional distance flown after re-intercepting the inbound course and flying back to the fix. In order to be consistent, we will assume that the aircraft's outbound heading will be the same for the next circuit. We can express the above statement with the following equation

$$\left|\operatorname{Cos}\theta_H - \bar{V}_W \operatorname{Cos}\alpha\right| \Delta t_{out} = \left|\operatorname{Cos}\sigma - \bar{V}_W \operatorname{Cos}\alpha\right| \Delta t_{in} \qquad (41)$$

Here, the left side of eq.(41) is the change in the distance covered in the x-direction on the outbound leg due to a change in Δt_{out} , whereas, the right hand side is the change in distance covered in the x-direction on the inbound leg due to a required change in Δt_{in} . The symbol $|\;|$ represents the absolute value of the quantity inside the vertical bars. Note that Δt_{in} can take on positive or negative values. Thus, we can solve for the required change in the outbound time, necessary to produce the required changed in the inbound time, i.e.

$$\Delta t_{out} = \frac{\left|\operatorname{Cos}\sigma - \bar{V}_W \operatorname{Cos}\alpha\right|}{\left|\operatorname{Cos}\theta_H - \bar{V}_W \operatorname{Cos}\alpha\right|} \Delta t_{in} \qquad (42)$$

As an example, consider the case where we have a pure headwind on the inbound leg to the holding fix. In this case,

$$\begin{aligned} \alpha &= 0 \\ \sigma &= 0 \\ \theta_H &= 180 \end{aligned} \qquad (43)$$

Since Cos(0)=1, and Cos(180)=-1, the change in the outbound time is given by

$$\Delta t_{out} = \frac{(1 - \bar{V}_W)}{(1 + \bar{V}_W)} \Delta t_{in} \qquad (44)$$

In the case of a windspeed ratio of $\bar{V}_W =$ 0.05, $\Delta t_{out} \simeq 0.9 \Delta t_{in}$, and thus for a required change in inbound time, one only needs 90 percent of that for the change in the outbound time. However, as the windspeed ratio increases to 0.2, we see that

$\Delta t_{out} = \frac{2}{3}\Delta t_{in}$, and thus, in order to make the required inbound time, the aircraft must correct the outbound time by only 2/3 of the required change in the inbound time. Clearly, the use of the approximation $\Delta t_{out} \simeq \Delta t_{in}$ would cause the pilot to fly additional circuits in order to obtain the required inbound time. Therefore, the rule-of-thumb for correcting the outbound time being used in the IFR community is only valid for small windspeed ratios. However, eq. (42) provides the exact ratio of the required change in outbound time for a prescribed change in inbound time, when the outbound heading is held constant.

In Section 2.2 we will investigate the two simplest cases, i.e., the direct headwind ($\alpha = 0$) and direct tailwind ($\alpha = 180$).

2.2 Holding Pattern Solution: Pure Headwind or Tailwind Case

The solution for the outbound time, given in eq. (15) contains the ratio of $\frac{\mathrm{Sin}\,\theta_H}{\mathrm{Sin}\,\sigma}$ and the value of the turn rate, k. In the pure headwind ($\alpha = 0$) or tailwind ($\alpha = 180$) cases, this ratio is indeterminate, i.e. $\frac{0}{0}$. The solution for this ratio must be obtained using a limiting process in elementary Calculus known as L'Hopital's rule, wherein the numerator and denominator are obtained using series expansions around the points $\alpha = 0$ and $\alpha = 180$.

In the pure headwind case ($\alpha = 0$, we can expand the a_i coefficients in eq. (23) around $\alpha = 0$, while retaining $\mathrm{Sin}\,\sigma$. We will designate these approximate coefficients with an overbar, i.e.

$$\begin{aligned} \bar{a}_1 &= 1 - (1 + \frac{6}{k\beta})\bar{V}_W \\ \bar{a}_2 &= \frac{6}{k\beta}\mathrm{Sin}\,\sigma \\ \bar{a}_3 &= -(1 - \bar{V}_W)\mathrm{Sin}\,\sigma \end{aligned} \qquad (45)$$

Equation (22) can be rewritten as

$$\bar{a}_1 \mathrm{Sin}\,\theta_H - \bar{a}_2 + \bar{a}_3 = 0 \qquad (46)$$

Since $\sigma = 0$, the relative outbound heading $\theta_H = 180$, and $\mathrm{Cos}(180) = -1$. Dividing eq. (46) by $\mathrm{Sin}\,\sigma$, we obtain the final equation for the ratio $\frac{\mathrm{Sin}(\theta_H)}{\mathrm{Sin}\,\sigma}$, i.e.

$$\frac{\sin\theta_H}{\sin\sigma} = \frac{[\frac{6}{k\beta} + 1 - \bar{V}_W]}{[1 - (1 + \frac{6}{k\beta})\bar{V}_W]} \qquad (47)$$

Note that although both $\sin\sigma = 0$ and $\sin\theta_H = 0$, the ratio of these quantities has a limit which is given by eq. (47). Substituting the above result into eq. (15) gives the final result for the outbound time t_{out}

$$t_{out} = \frac{60\beta[1 - (1 + \frac{6}{k\beta})\bar{V}_W]}{(1 + \bar{V}_W)} \qquad (48)$$

In the pure tailwind case we can perform a similar expansion around $\alpha = 180$ degrees, however, we can also obtain the result quicker by redefining the tailwind as $\alpha = 0$, and replacing $\bar{V}_W$ with -$\bar{V}_W$. Thus, the Sine ratio and t_{out} in the pure tailwind case become

$$\frac{\sin\theta_H}{\sin\sigma} = \frac{[\frac{6}{k\beta} + 1 + \bar{V}_W]}{[1 + (1 + \frac{6}{k\beta})\bar{V}_W]} \qquad (49)$$

$$t_{out} = \frac{60\beta[1 + (1 + \frac{6}{k\beta})\bar{V}_W]}{(1 - \bar{V}_W)} \qquad (50)$$

In the pure headwind case, eq. (48) shows the outbound time goes to zero when

$$\bar{V}_W = \bar{V}^*_W = \frac{1}{(1 + \frac{6}{k\beta})} \qquad (51)$$

For values of $\bar{V}_W > \bar{V}^*_W$, the value of t_{out} becomes negative and indicates a failure in the analysis. However, the reason for the failure is due to the fact that when $\bar{V}_W > \bar{V}^*_W$, the aircraft must fly outbound along the inbound course (i.e. along the positive x axis) for a specified time. This case corresponds to the value of $\theta_H = 0$. Thus, when $\bar{V}_W > \bar{V}^*_W$, the term $\frac{\sin\theta_H}{\sin\sigma}$ is no longer obtained by solving eq. (46), but by solving

$$\bar{a}_1 \sin\theta_H + \bar{a}_2 + \bar{a}_3 = 0 \qquad (52)$$

Where we have replaced Cos(180)=-1 with Cos(0)=1. If we now solve for the ratio $\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma}$ in eq. (52), we obtain

$$\frac{\text{Sin}\,\theta_H}{\text{Sin}\,\sigma} = \frac{-(\frac{6}{k\beta})(1-\bar{V}_W)}{1-(1+\frac{6}{k\beta})\bar{V}_W} \quad (53)$$

Substituting eq. (53) into eq. (15), we obtain the following equation for the outbound time when $\bar{V}_W \geq \bar{V}_W^*$

$$t_{out} = 60\beta \frac{[(1+\frac{6}{k\beta})\bar{V}_W - 1]}{(1-\bar{V}_W)} \qquad (54)$$

Summarizing the outbound times in the headwind case

$$t_{out} = \frac{60\beta[1-(1+\frac{6}{k\beta})\bar{V}_W]}{(1+\bar{V}_W)} \qquad \bar{V}_W \leq \frac{1}{(1+\frac{6}{k\beta})} \qquad (55)$$

$$t_{out} = 60\beta \frac{[(1+\frac{6}{k\beta})\bar{V}_W - 1]}{(1-\bar{V}_W)} \qquad \bar{V}_W \geq \frac{1}{(1+\frac{6}{k\beta})} \qquad (56)$$

As an example, in the case of an aircraft performing a standard rate turn in the holding pattern (k=3 degrees/sec), and using a required inbound time of one minute, corresponding to a holding pattern below 14000MSL, results in the following equations for t_{out}:

Headwind case:

$$t_{out} = \frac{60[1-3\bar{V}_W]}{(1+\bar{V}_W)} \qquad \bar{V}_W \leq \frac{1}{3} \qquad (57)$$

$$t_{out} = 60\frac{[(3\bar{V}_W - 1]}{(1-\bar{V}_W)} \qquad \bar{V}_W \geq \frac{1}{3} \qquad (58)$$

Tailwind case:

$$t_{out} = 60\frac{(1+3\overline{V}_W)}{(1-\overline{V}_W)} \qquad (59)$$

Although eqs (57) and (59) were derived using a limiting process, we can obtain these equations using physical arguments. For example, in the headwind/tailwind case, the OWCA is identically zero. Under no wind conditions, the aircraft reaches the holding fix (x=0) and turns 180 degrees. At the end of the turn the aircraft will also be at x=0. In the case of a headwind on the inbound leg, after the first 180-degree turn the aircraft will have drifted a distance downwind by the amount V_W times one minute. On the outbound leg, aircraft travels a distance equal to (V_{TAS} +V_W)*t_{out}. After the final 180-degree turn, the aircraft will have drifted an additional distance downwind equal to V_W times one minute and will have intercepted the inbound course. The aircraft is now located a distance $2V_W + (V_{TAS} + V_W)t_{out}$ downwind of the fix. Since the aircraft must return to the fix in one minute, we see that the following equation must hold

$$(V_{TAS} - V_W)*1 = 2V_W + (V_{TAS} + V_W)t_{out} \qquad (60)$$

Here t_{out} is in minutes. Solving eq. (60) for t_{out} we obtain the following equation

$$t_{out} = \frac{(1-3\overline{V}_W)}{(1+\overline{V}_W)} \qquad (61)$$

Multiplying eq. (61) by 60 gives us the outbound time in seconds, i.e.

$$t_{out} = 60\frac{(1-3\overline{V}_W)}{(1+\overline{V}_W)} \qquad (62)$$

Note that eq. (57) and (62) are identical, verifying the limiting process gives us the correct answer. Again, substituting -V_W for V_W, gives the correct answer for the tailwind case on the inbound leg as shown in eq. (59)

In the case of a pure headwind, with $\overline{V}_W = \frac{1}{3}$ the outbound time is identically zero. In this scenario, the aircraft reaches the holding fix, performs a 360 degree turn re-intercepting the inbound course, and then flies one minute to the holding fix. Under this wind condition, the time to fly the holding pattern is exactly three minutes. This particular holding pattern is shown in Figure 4 below.

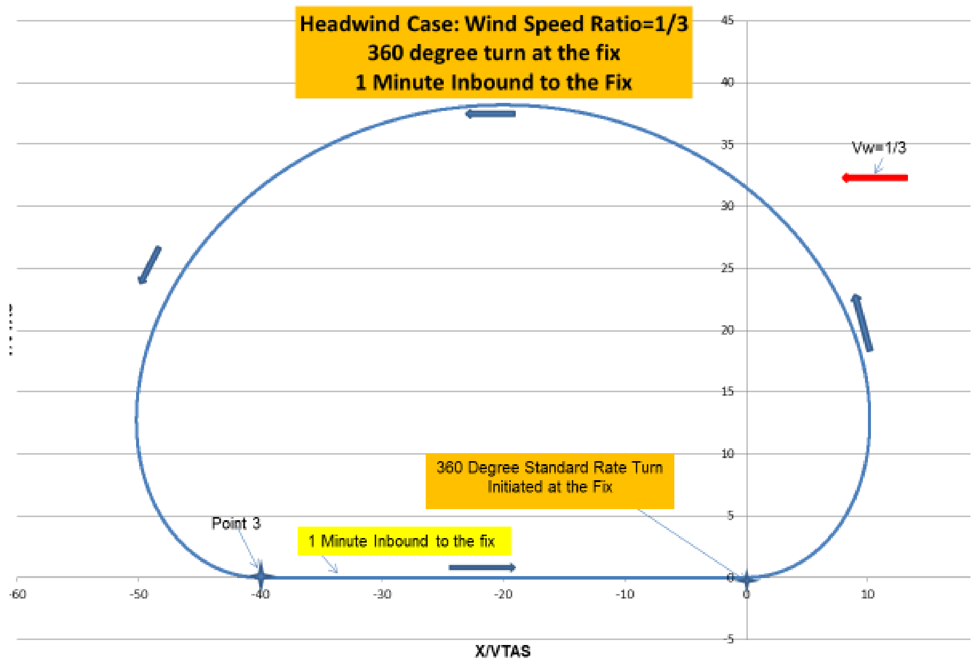


Figure 4: Case $\bar{V}_W = \frac{1}{3}$, T_{OUT}=0

In order to understand this particular holding pattern, consider the aircraft reaching the holding fix and executing a two-minute standard rate turn. After re-intercepting the inbound course, the aircraft has been blown downwind a distance V_W *(2 minutes). If the aircraft flies for an additional minute while on the headwind, the aircraft will be blown an addition distance downwind equal to V_W*(1 minute). Thus, during these three minutes, the aircraft has been blown V_W*(3 minutes) downwind from the fix. In order for the aircraft to arrive at the fix at the end of 3 minutes, the aircraft's TAS must be three times the windspeed. Thus, this particular case corresponds to a value of $\bar{V}_W = \frac{1}{3}$. If $\bar{V}_W > \frac{1}{3}$, it will be impossible to fly a one-minute inbound leg to the holding fix, unless the aircraft tracks outbound along the inbound course (i.e. along the positive x-axis) for a given amount of time before making a 360 turn to re-intercept the inbound course. The outbound time in this case is given by eq. (58). One can also obtain this result using physical arguments. For example, if the aircraft is performing a standard rate turn (i.e. k=3) and starts a two-minute turn when it reaches the holding fix, it will have been blown downwind a distance 2*V_W. After intercepting the inbound radial, it travels a distance of $(V_{TAS} - V_W)\beta$ toward the fix. The x-location of the aircraft at this time is given by $-(2+\beta)V_W + V_{TAS}\beta$. If $\bar{V}_W > 1/(1+2/\beta)$, the aircraft will need to fly a distance beyond the fix equal to $(V_{TAS} - V_W)t_{out}$. In order to arrive at the holding fix after β minutes, the following equation must hold

$$-(2+\beta)V_W + V_{TAS}\beta + (V_{TAS} - V_W)t_{out} = 0 \qquad (63)$$

Solving for t_{out}

$$t_{out} = 60\frac{[(2+\beta)\bar{V}_W - \beta]}{(1-\bar{V}_W)} \qquad (64)$$

We observe that eq. (64) is identical to eq. (56), which again, confirms the results of the limiting process.

As an example, if $\bar{V}_W = 0.4$ and β =1, the outbound time t_{out} is 20 seconds. Thus, if the aircraft reaches the fix and tracks outbound for 20 seconds, performs a 360 standard-rate turn, and flies for one minute, the aircraft will be at the holding fix at the end of the one-minute inbound leg. Instead of using eq. (58) the Pilot can use the following simple method: When $\bar{V}_W > \frac{1}{3}$, the Pilot can track the inbound course to the holding fix, perform a 360-degree standard-rate turn, re-intercept the inbound course, and then time the inbound leg to the holding fix. The time to fly beyond the fix, t_{out}, is just the difference between the time for the inbound leg and the required inbound time of either one-minute or one-minute and 30 seconds.

In the pure headwind or tailwind case, both the IWCA and OWCA are zero, and thus,

$$\frac{\operatorname{Sin}\theta_H}{\operatorname{Sin}\sigma} = \frac{\operatorname{Sin}\delta}{\operatorname{Sin}\sigma} \simeq \frac{OWCA}{IWCA} \qquad (65)$$

Where $\delta = 180 - \theta_H$ and corresponds to the OWCA. In the case of $\bar{V}_W < \frac{1}{3}$ it is easy to see that the M-Factor is bounded by

$$\frac{(3-\bar{V}_W)}{(1-3\bar{V}_W)} \geq \frac{OWCA}{IWCA} \geq \frac{(3+\bar{V}_W)}{(1+3\bar{V}_W)} \qquad (66)$$

Equation (66) shows that the maximum value of the M-Factor corresponds to the direct headwind case, and the minimum value of the M-Factor corresponds to the direct tailwind case. In the case of $\bar{V}_W = 0.3$, the M-Factor is bounded by

$$27 \geq \frac{OWCA}{IWCA} \geq 1.74 \qquad (67)$$

Equation (66) debunks many of the articles in the open literature which state that the M-Factor is always between 2 and 3.

In Section 2.3, we investigate the arbitrary wind case where the outbound turn is more than 90 degrees (i.e., the standard Type-1 holding pattern).

2.3 Holding Pattern with an Arbitrary Wind (Type-1)

The general solution of the "Holding Pattern" problem is given by eqs. (1), (15), (22) and (23), which are solved for the IWCA, σ, the outbound time, t_{out}, and outbound heading θ_H, given the coefficients a_1, a_2, and a_3. Again a_1-a_3 are functions of the windspeed ratio $\bar{V}_W$, and the wind direction α, relative to the inbound course. In addition, a_1 and a_2 are also functions of both the turn rate k and the inbound time to the fix β. The latest AIM (paragraph. 5-3-8) recommends that the OWCA should be 3 times the IWCA in order to properly re-intercept the inbound course to the holding fix. We will now show that this rule-of-thumb for the OWCA is useful only under a limited set of wind conditions. In order to show this important conclusion, we will determine the M-Factor, which is the ratio of the outbound OWCA to the IWCA. The OWCA, δ, is related to the relative outbound heading by the following equation

$$\delta = 180 - \theta_H \qquad (68)$$

We will now calculate the "Holding Pattern" solution for the following range of wind conditions

$$\begin{aligned} 0 \le \bar{V}_W \le 0.3 \\ 0 \le \alpha \le 180 \end{aligned} \qquad (69)$$

For GA aircraft holding at speeds of 100-110 KTAS, the maximum wind speed would correspond to 30-33 knots. As was discussed earlier, the "Holding Pattern" solution for negative values of α are obtained from the solutions for the positive values of α as follows

$$\begin{aligned} \sigma(\bar{V}_W, -\alpha) &= -\sigma(\bar{V}_W, \alpha) \\ \theta_H(\bar{V}_W, -\alpha) &= -\theta_H(\bar{V}_W, \alpha) \\ t_{out}(\bar{V}_W, -\alpha) &= t_{out}(\bar{V}_W, \alpha) \end{aligned} \qquad (70)$$

We have chosen a maximum value of $\bar{V}_W = 0.3$ in order to avoid the Type-2 holding patterns that arise when the windspeed becomes greater than $\frac{1}{3}$ for the case of the one-minute inbound leg. We will address the Type-2 "Holding Pattern" solution later in this Section. In addition, as pointed out earlier, the shape of the holding pattern is determined by $\bar{x}$ and $\bar{y}$, however the actual dimensions of the holding pattern are determined by multiplying these quantities by the term $\frac{V_{TAS}}{\omega}$, where ω is given by eq.(14). In order to provide a database for typical GA aircraft flying a Type-1 holding patterns, we will assume the value of k=3 degrees/sec and β =1 (one-minute inbound leg). However, we can easily develop another set of curves for $\beta = \frac{3}{2}$, which corresponds to a holding pattern flown at or above 14000MSL.

In the case of the classical Type-1 holding pattern, wherein the outbound turn is greater than 90 degrees relative to the inbound course, we have provided the solution for six values of the windspeed ratio: 0.05, 0.1, 0.15, 0.2, 0.25, and 0.3. Figures 5-7 show the IWCA, the M-Factor (i.e. OWCA/IWCA), the outbound time, $t_{out,}$ measured from the point at which the aircraft has turned to the outbound heading, θ_H , and the OWCA.

We will now discuss some important conclusions that can be reached by reviewing Figures 5-7. First, the maximum IWCA will occur when $\alpha = 90$ degrees. In this case the IWCA is given by

$$\sigma = \mathrm{Sin}^{-1}(\bar{V}_W) \qquad (71)$$

Figure 5 shows the IWCA for the six values of the windspeed ratio. As can be seen, the peak value occurs at $\alpha = 90$ degrees, with the maximum being 17.5 degrees corresponding to the value $\bar{V}_W = 0.3$. In addition, excluding $\alpha = 90$, there are two values of α that produce the same value of the IWCA. One corresponds to a headwind component, and the other corresponds to a tailwind component

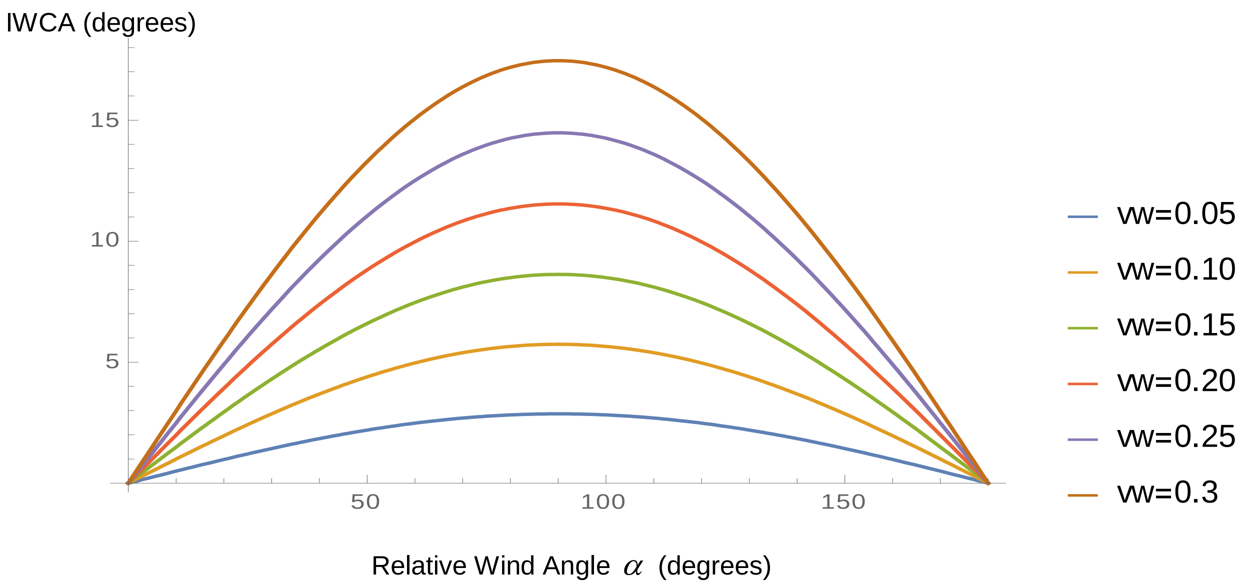


Figure 5: Magnitude of the IWCA versus Relative Wind Angle α for Values of $\bar{V}_W$

Figure 6a and 6b show the M-Factor versus relative wind angle for the six values of the windspeed ratio. It is easy to see that the AIM recommendation of using an M-Factor of 3 on the outbound leg is valid only under limited conditions. In Figure 6b, we have annotated a Cyan rectangle where the M-Factor is bounded between 2.5 and 3.5. In this limited region, using an M-Factor of 3 would be a good approximation. This region is defined by $70 \le \alpha \le 95$ for $\bar{V}_W \le 0.3$. However, if $\bar{V}_W \le 0.05$, using an M-Factor of 3 would also be a good approximation over the entire range of α. Note that as $\alpha \to 0$ or $\alpha \to 180$,

$$\text{M-Factor}=\frac{\delta}{\sigma} \approx \frac{\operatorname{Sin}\theta_H}{\operatorname{Sin}\sigma} \qquad (72)$$

Thus, when $\alpha \to 0$, the M-Factor reaches its maximum value given by $\frac{(3-\bar{V}_W)}{(1-3\bar{V}_W)}$, and as $\alpha \to 180$, the M-Factor reaches its minimum value given by $\frac{(3+\bar{V}_W)}{(1+3\bar{V}_W)}$.

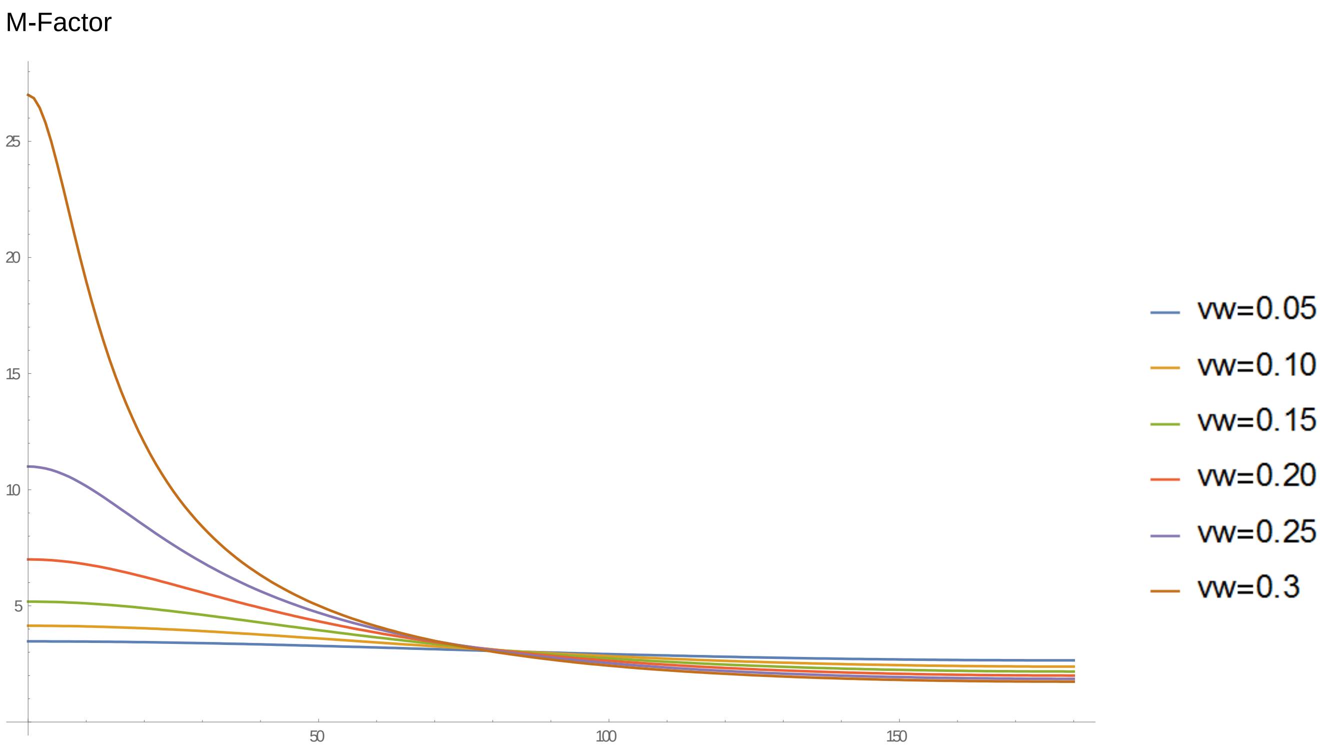


Figure 6a: M-factor versus Relative Wind Angle α for Various Values of $\bar{V}_W$

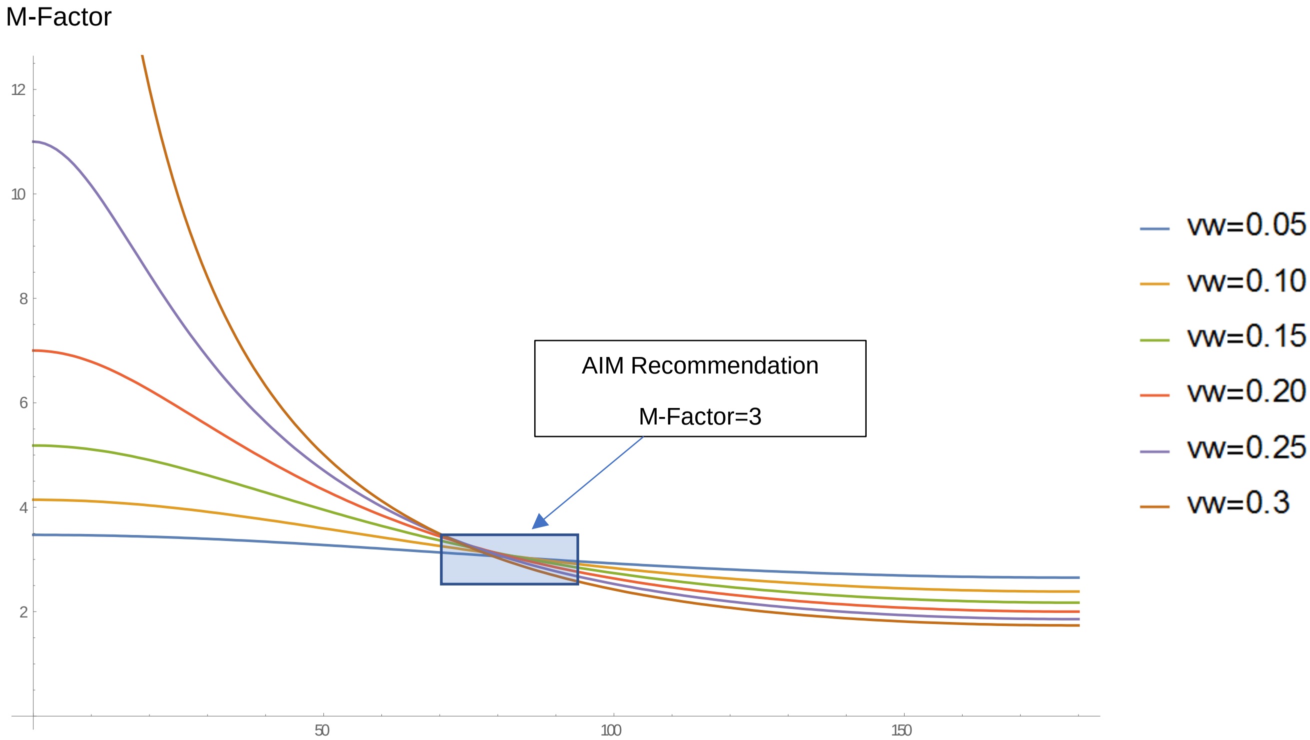


Figure 6b: M-Factor versus Relative Wind Angle α (Expanded Scale)

Figure 7a shows the outbound time measured from the point that the aircraft has turned to the required outbound heading θ_H . Note that as the windspeed ratio approaches $\frac{1}{3}$, with $\alpha \rightarrow 0$ the outbound time approaches zero. This is consistent with the limiting case of the headwind discussed previously. Clearly, one would like to keep the windspeed ratio at or below 0.25 in order for the IFR Pilot to have a reasonable amount of time to fly the outbound leg before turning inbound to re-intercept the inbound course.

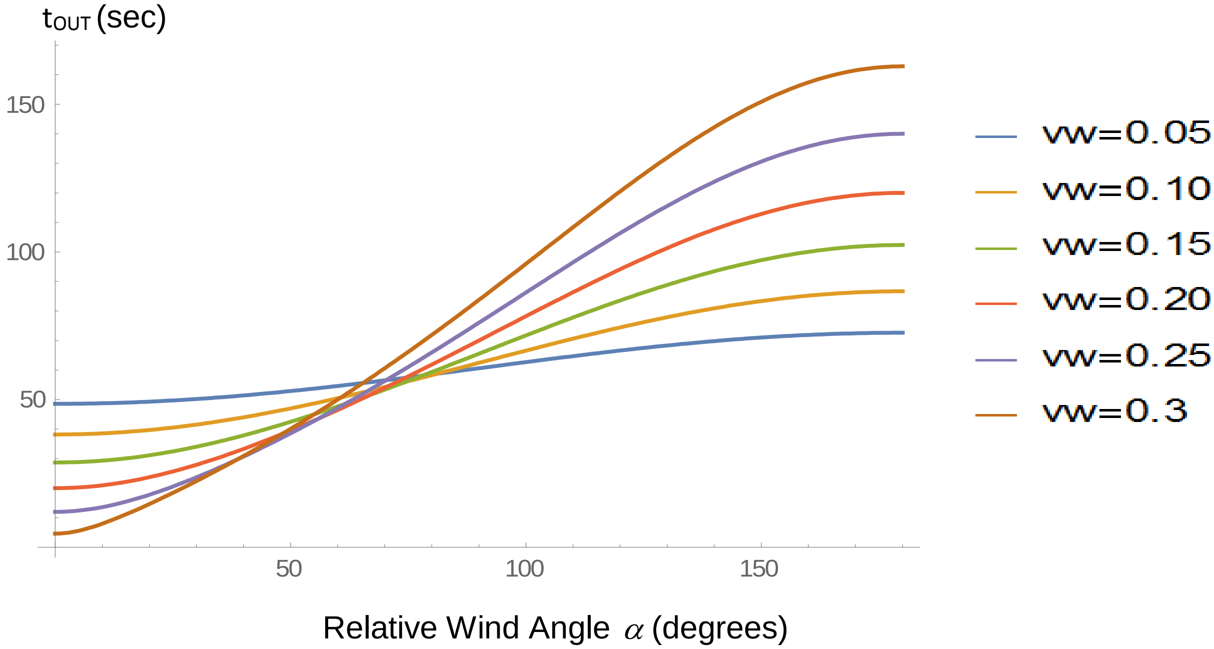


Figure 7a: Required Outbound Time versus Relative Wind Angle α for Various Values of $\bar{V}_W$

Reference 1 describes a rule of thumb for correcting the outbound time as a function of the windspeed. It states "decrease the outbound time 2 seconds per 1 knot of tailwind on the outbound leg (i.e. a headwind on the inbound leg) and increase the outbound time 2 seconds per 1 knot of headwind on the outbound leg (i.e. a tailwind on the inbound leg). Figure 7a shows the required outbound time for various windspeed ratios. These ratios are shown for an increment of $\bar{V}_W = 0.05$. Note that gradient of the outbound time with respect to the windspeed ratio is not the same for the headwind and the tailwind cases. The tailwind case on the inbound leg has a larger gradient in outbound time for a given change in the windspeed ratio when compared to the headwind case on the inbound leg. In addition, the outbound time gradient is also a function of the aircraft V_{TAS}, whereas, Reference 1 shows the rule-of-thumb is independent of the aircraft V_{TAS}. Using eq. (57) for the case of a headwind on the inbound leg, and eq. (59) for the case of a tailwind on the inbound leg, we can express the outbound time gradients for the headwind and tailwind cases as

$$\left(\frac{dt_{out}}{dV_W}\right)_{HW} = \frac{-240}{V_{TAS}(1+\bar{V}_W)^2}$$
$$\left(\frac{dt_{out}}{dV_W}\right)_{TW} = \frac{240}{V_{TAS}(1-\bar{V}_W)^2} \qquad (73)$$

These gradients are clearly a function of the windspeed ratio. Note, for V_{TAS}=100 knots, the outbound time gradient is given by

$$(\frac{dt_{out}}{dV_W})_{HW} = \frac{-2.4}{(1+\bar{V}_W)^2}$$
$$(\frac{dt_{out}}{dV_W})_{TW} = \frac{2.4}{(1-\bar{V}_W)^2} \quad (74)$$

When $\bar{V}_W = 0.2$, the outbound time gradient on a headwind is 1.67 seconds/knot, whereas on a tailwind, the outbound time gradient is 3.75 seconds/knot. Clearly, using a constant value for both headwind and tailwind is not correct.

Figure 7b shows the OWCA versus the relative wind angle α. Note that the OWCA is determined by

OWCA=M-Factor*IWCA (75)

It is important to understand that although the M-Factor is a maximum at $\alpha = 0$, the IWCA is zero and thus, the OWCA is identically zero on the headwind. It is easy to see that the OWCA reaches a maximum somewhere between $0 \le \alpha \le 90$ degrees. However, as the windspeed ratio increases, the peak shifts toward $\alpha = 0$. Clearly the recommendation of an M-Factor of 3 completely misses the large OWCA at both the higher values of $\bar{V}_W$ and when $\alpha < 70$ degrees.

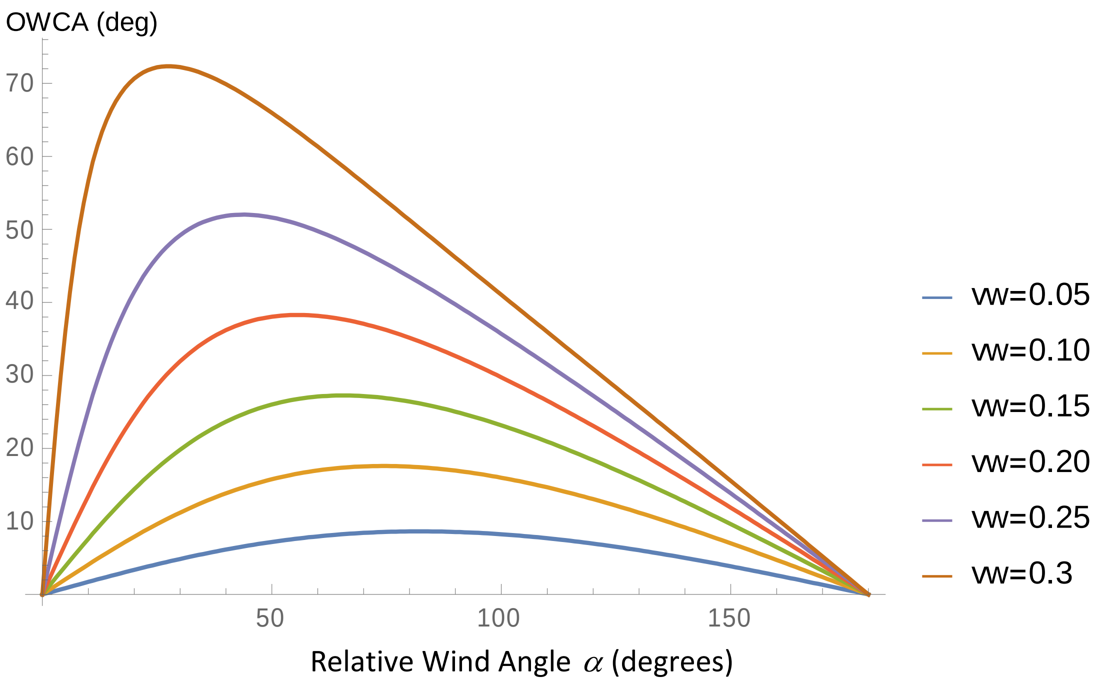


Figure 7b: OWCA versus Relative Wind Angle α for Various Values of $\bar{V}_W$

Figure 8 shows a comparison of the holding patterns for $\alpha = 45$ and 135 degrees, corresponding to a windspeed ratio of 0.3. Note the significant difference in the OWCA between the two cases. In the case of a tailwind component on the inbound course, the aircraft must be flown further outbound in order to meet the required one-minute inbound leg. In order to prevent the aircraft from undershooting the inbound course, the OWCA must be smaller. Since the IWCA will be the same in both cases, it can easily be seen that the M-Factor in the tailwind case will be smaller than in the headwind case.

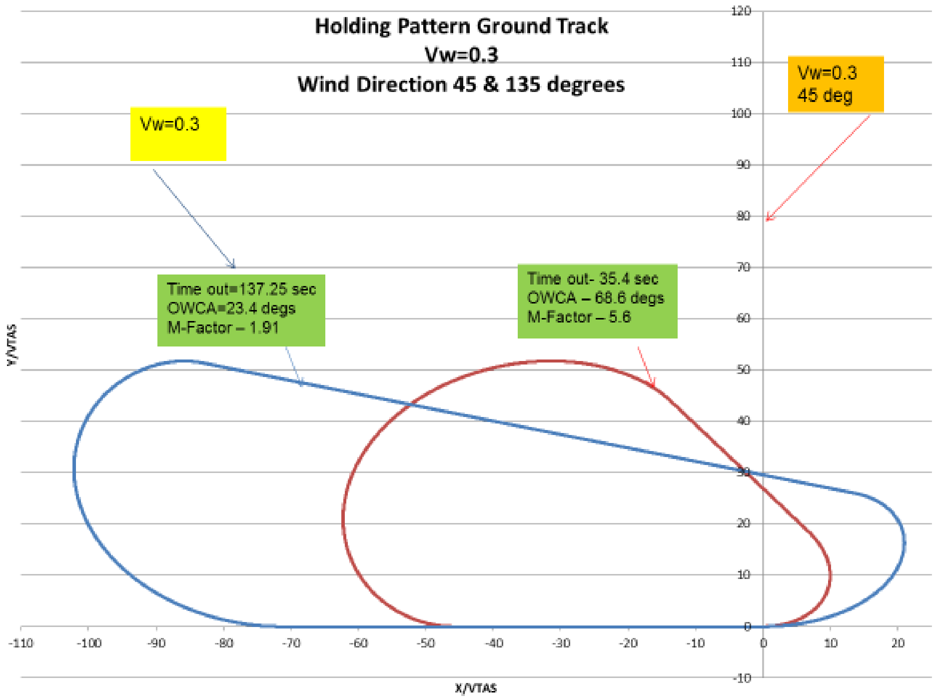


Figure 8: Comparison of M-Factor for $\bar{V}_W = 0.3, \alpha = 45$ and $\alpha = 135$

As an example of a Type-1 holding pattern, we consider the case where $\alpha = 45$ degrees and $\bar{V}_W = 0.3$. Figure 9a shows the normalized x-y coordinates of the holding pattern, with the outbound time and outbound heading shown on the Figure. Note that the headings that are annotated on the holding pattern now correspond to the heading that would be indicated on the aircraft Heading Indicator (HI). Since on the outbound leg the aircraft is turned 111.5 degrees from the inbound course, this would correspond to the HI reading 248.5 degrees when the inbound course is 360 degrees. Here the outbound time, measured from the point at which the aircraft turns to the outbound heading, is 35.4 seconds. In this case the IWCA is 12.2 degrees, so the M-Factor for this case is

5.6 (i.e. $\frac{(180-111.5)}{12.2}$), which is nearly twice the AIM recommendation for the M-Factor. We have tested this particular “Holding Pattern” solution on a Frasca 131 FTD. The results of the flight simulation were that (1) The aircraft re-intercepted the inbound course with the CDI centered and (2) The aircraft reached the holding fix in 61 seconds, which confirms the “Holding Pattern” solution previously derived.

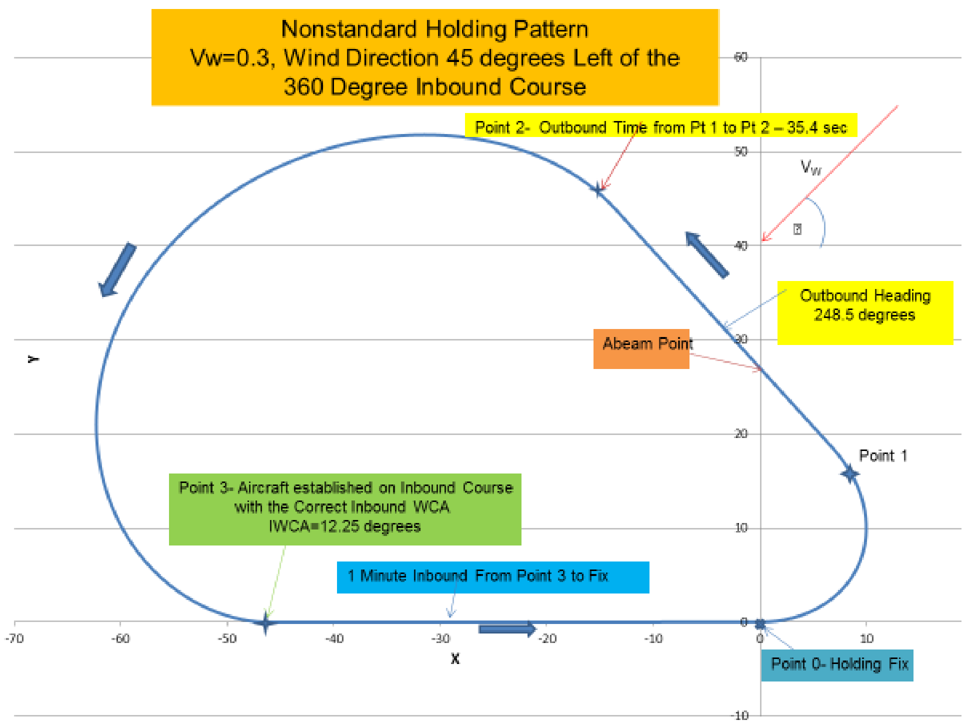


Figure 9a: Holding Pattern Solution for $\bar{V}_W = 0.3, \alpha = 45, \alpha = 135$, b=1, k=3

Although we have shown solutions for $0 \le \alpha \le 180$ and windspeed ratios up to 0.3, we will now consider the case of the direct crosswind in the holding pattern. This particular case has a very simple closed form solution for both the outbound heading and outbound time. In the crosswind case, $\alpha = 90$ degrees and thus, Cos(90)=0, and Sin(90)=1. Thus, the a_i coefficients in eq.(23) become

$$a_1 = \text{Cos}\,\sigma$$
$$a_2 = \frac{6}{k\beta}\text{Sin}\,\sigma \qquad (76)$$
$$a_3 = -\text{Cos}\,\sigma\,\text{Sin}\,\sigma$$

Substituting eq. (76) into eq. (26) gives the following result θ_H

$$\mathrm{Cos}\,\theta_H = -\frac{(1-\bar{V}_W^{\,2})^{\frac{3}{2}}}{(1+3\bar{V}_W^{\,2})} \qquad (77)$$

Using eq.(16), we see that the outbound time in seconds is given by

$$t_{out} = 60\frac{(1+3\bar{V}_W^{\,2})}{(1-\bar{V}_W^{\,2})} \qquad (78)$$

We can compute the outbound time gradient for the crosswind case using eq. (78). The final result is given by the following equation

$$\left(\frac{dt_{out}}{dV_W}\right)_{xwind} = \frac{480\bar{V}_W}{V_{TAS}(1-\bar{V}_W^{\,2})^2} \qquad (79)$$

In the case $\bar{V}_W = 0.2$ and a value of V_{TAS}=100 knots, the outbound time gradient is 1.04 seconds/knot. Again, using a constant value of the outbound time gradient for all values of the relative wind would not be correct.

If we expand eq. (77) in the limit of $\bar{V}_W \to 0$, we see that the M-Factor approaches 3, and thus, in this limit, the OWCA$\to$ 3*IWCA. Figure 9b shows the ground tracks for the crosswind case for values of $\bar{V}_W$ =0.1, 0.2, and 0.3. It is easy to show by series expansion methods, that for arbitrary α, in the limit of $\bar{V}_W \to 0$, the M-Factor$\to$3. This result can be seen in Figure 6b for small values of $\bar{V}_W$.

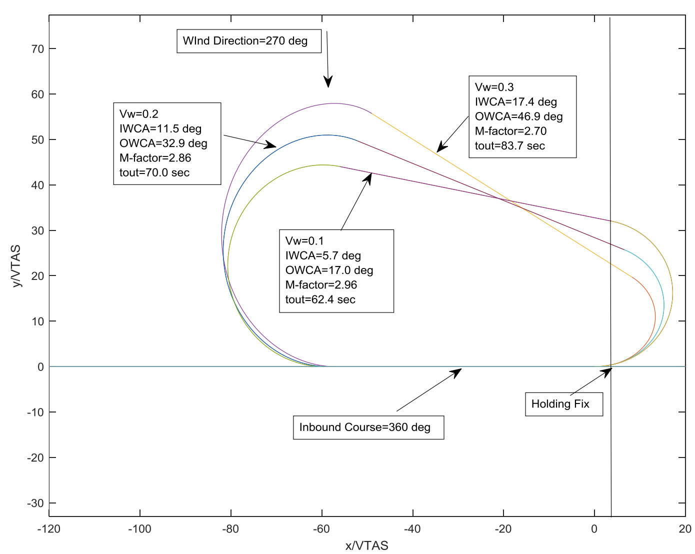


Figure 9b: Holding Pattern Ground Track for Direct Crosswind (k=3, β=1)

For those interested in understanding why the M-Factor is 3 in the limit of $\bar{V}_W \to 0$ we will consider the holding pattern with a one-minute inbound leg. In this limit we assume the outbound time is close to one minute. Since the time for the two turns add up to two minutes, the total time for the three legs is 3 minutes. The component of the wind perpendicular to the inbound course is given by $V_W \operatorname{Sin}\alpha$. Thus, during the 3 minutes, the aircraft will drift a distance of $3V_W \operatorname{Sin}\alpha$ perpendicular to the inbound course. Since this distance can only be made up on the outbound leg, the OWCA, δ, can be expressed by

$$\operatorname{Sin}\delta \simeq \frac{3V_W \operatorname{Sin}\alpha}{V_{TAS}} = 3\bar{V}_W \operatorname{Sin}\alpha \qquad (80)$$

Using the IWCA given in eq. (1), we see that

$$\frac{\operatorname{Sin}\delta}{\operatorname{Sin}\sigma} = \frac{3\bar{V}_W \operatorname{Sin}\alpha}{\bar{V}_W \operatorname{Sin}\alpha} = 3 \qquad (81)$$

Note, for small values of both the IWCA and OWCA, eq. (81) can be written as

$$\frac{\operatorname{Sin}\delta}{\operatorname{Sin}\sigma} \approx \frac{\delta}{\sigma} = 3 \qquad (82)$$

The above argument clarifies the origin of factor of 3 between the OWCA and the IWCA, and shows the factor of 3 only applies to the case of the weak-wind limit.

Prior to the late 1980's, the recommendation was to use a factor of 2 between the OWCA and the IWCA. This was based on the incorrect assumption that the Pilot only needed to correct the wind drift during the 2 minutes of turning flight. Note that if we substitute eq. (82) into eq. (16), and using the fact that the $\operatorname{Sin}\delta = \operatorname{Sin}\theta_H$, confirms that in this limit, the outbound time is one minute. In addition, there have been numerous articles and forums discussing the bounds on the M-Factor. Many of these state that the M-Factor is bounded between 2 and 3. Bounding the M-Factor between 2 and 3 is a myth and is due to a lack of understanding of how the wind affects the OWCA. The true variation of the M-Factor is shown in Figures 6a and 6b.

Finally, Figure 9c, shows an example of determining the "Holding Pattern solution for the case of a 4 nm inbound leg, instead of a minute inbound leg. Here, we use the identical wind speed ratio and direction as in Figure 9a. In both cases the TAS for is 100 knots. The required value of the inbound time for the 4 nm inbound leg is 3.14 minutes.

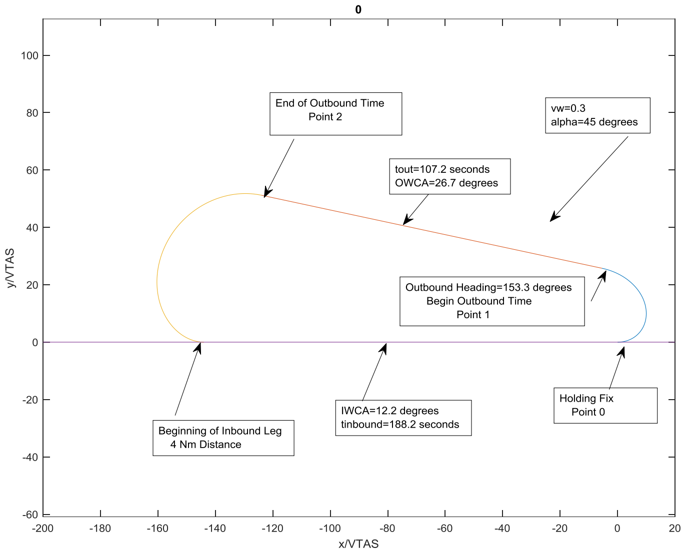


Figure 9c: Holding Pattern with 4 nm Inbound Leg ($\bar{V}_W$ =0.3, $\alpha = 45$ deg, k=3, $\beta = 3.14$)

2.4 Holding Pattern with an Arbitrary Wind (Type-2)

Although Type-2 holding patterns will be encountered in strong wind conditions with the inbound course being flown on a headwind component, it is important for all IFR Pilots to understand the complexity of this type of holding pattern. In this case, the outbound time can control the undershoot or overshoot of the interception of the inbound course, whereas, the outbound heading can control the inbound time. This is completely contrary to the way we fly Type-1 holding patterns.

Using Figure 3, which shows the boundary between the Type-1 and Type-2 holding patterns in $\bar{V}_W - \alpha$ space, we select the value $\alpha = 30$ degrees and the value $\bar{V}_W = 0.4$. Clearly this wind combination would shift the aircraft into the Type-2 holding pattern region. The solution of this Type-2 holding pattern is shown in Figure 10. In this particular holding pattern the IWCA is 11.5 degrees. However, in this case, the aircraft relative outbound heading is 63.3 degrees from the inbound course and is flown on this heading for 34.6 seconds. The aircraft then turns to a relative heading of 348.5 degrees (i.e. 360-11.5), re-intercepts the inbound course and flies exactly one minute to reach the holding fix. This Type-2 holding pattern was simulated on a Frasca 131 FTD with the following results: (1) The aircraft re-Intercepted the inbound course with the CDI centered and (2) The inbound time was 59 seconds, thereby validating the “Holding Pattern” solution.

It is important to point out the completely different shape of this holding pattern compared to the Type-1 holding pattern previously shown. To the author's knowledge, this type of holding pattern has never been discussed in the open literature. One can easily see that the outbound heading is controlling the inbound time, whereas, the outbound time is controlling the undershoot/overshoot of the inbound course. This is completely opposite to the way we normally fly the Type-1 holding pattern. Clearly, without prior knowledge of this type of holding pattern, the IFR Pilot would spend a considerable amount of time trying to get the one-minute inbound time correct, and in the authors opinion, even seasoned IFR Pilots would not be able fly this pattern correctly.

As an example, in Figure 11 we show the aircraft track on the first circuit when flying the holding pattern using the AIM recommendations for the OWCA. We will assume the IFR Pilot has intercepted the inbound course and has determined the IWCA to be 11.5 degrees for these wind conditions (i.e., $\bar{V}_W = 0.4$ and $\alpha = 30$ degrees). Using an M-factor of 3, the Pilot turns the aircraft outbound to a relative heading of 145.5 degrees from the inbound course (360 degrees), which puts the aircraft on a heading of 214.5 degrees. The Pilot then flies for 60 seconds between points 1 and 2, and then turns the aircraft to a heading of 360 degrees plus the IWCA, i.e. the aircraft heading is 348.5 degrees. At this time, the Pilot observes that the aircraft has overshot the inbound course by an amount $\frac{\Delta y}{V_{TAS}} = 1.92$. If the TAS of the aircraft is 90 knots, the overshoot is only about 292 feet. Note that we need not complete the segment from point 3 to 0, since the aircraft at this point in time, will fly parallel to the x-axis and provide an accurate inbound time to the holding fix. This time would be identical to the above inbound time, had the aircraft re-intercepted the inbound course and flew directly to the fix. The Pilot then has a choice of flying directly to the fix or turning the aircraft slightly to the left and re-intercept the inbound course. If the Pilot flies directly to the fix, the inbound time to the fix will be approximately 176.5 seconds. This is nearly a 3-minute inbound leg. The Pilot's first thought is to increase the OWCA slightly because of overshooting the inbound course. However, the Pilot must shave off nearly 2 minutes in order to meet the required one-minute inbound time to the fix. The Pilot may realize that he/she needs to fly past the fix and turn less than 90 degrees, but there are no guidelines as the required heading and the outbound time. At this point in the game, the Pilot becomes confused and frustrated. As a point of interest, in this type of holding pattern, there is no way to identify the abeam point for starting the outbound time. In fact, over 99% of the IFR Pilot population would not be able to correctly fly this Type-2 holding pattern.

The key point for all IFR Pilots to understand is not to be forced into flying a Type-2 holding pattern, i.e. the Pilot should increase the aircraft's TAS such that the windspeed ratio is less than $\frac{1}{3}$. In fact, keeping the windspeed ratio below 0.25 will allow the Pilot to fly the familiar Type-1 holding pattern with a sufficient amount of outbound time before turning to re-intercept the inbound course and achieve the one-minute inbound time.

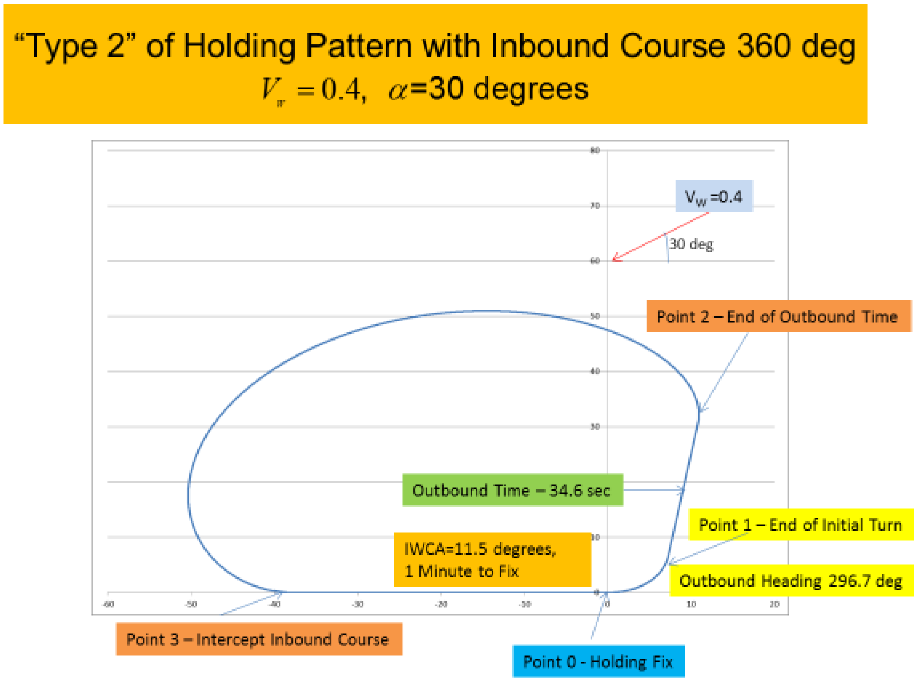


Figure 10: Type-2 Holding Pattern: $\bar{V}_W = 0.4$, $\alpha = 30$ degrees

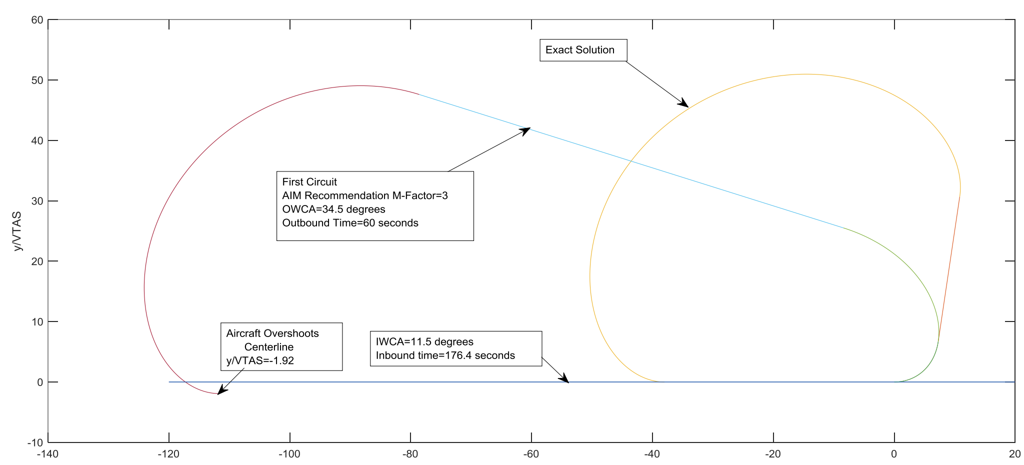


Figure 11: Type-2 Holding Pattern: $\bar{V}_W = 0.4$, $\alpha = 30$ degrees, β =1, k=3

Comparison of Exact Solution with AIM Recommendations for First Circuit

In Section 3 we will discuss techniques that can be used when flying a Type-1 holding pattern that will allow the IFR Pilot to converge to the “Holding Pattern” solution in a minimum number of circuits.

3.0 Proper Technique for Flying the Type-1 Holding Pattern

Up to this point we have discussed the Type-1 and Type-2 holding patterns. We have recommended flying the holding patterns below a windspeed ratio of 0.25 to avoid the dreadful Type-2 holding pattern. In this Section, we will concentrate on flying the Type-1 holding pattern.

In general, IFR Pilots undergoing training in the area of holding patterns are instructed to fly the initial holding pattern by determining the IWCA and multiplying its value by 3 to obtain the OWCA. The initial outbound leg is flown for either 60 seconds or 90 seconds, depending on the aircraft altitude. From then on, all corrections are made in the following manner: (a) Timing: Decrease the outbound time when the inbound time is greater than the prescribed inbound time and increase the outbound time when the inbound time is less than the prescribed inbound time; and (b) Wind Correction: Increase the OWCA when overshooting the inbound course and decrease the OWCA when undershooting the inbound course. This process is one of “Trial and Error” in attempting to converge to the “Holding Pattern” solution. Recall, the converged solution occurs when the aircraft meets the required inbound time with the aircraft intercepting the inbound course at the time the aircraft’s heading is equal to that of inbound course plus the IWCA. In fact, in Section 4, we will show this method is very inefficient in converging to the correct “Holding Pattern” solution. We will now show that using a formal systematic process will actually converge to the “Holding Pattern” solution in a minimum number of circuits. This methodology is developed below.

The exact solution to the “Holding Pattern” problem allows us to obtain a wealth of information on how to converge the holding pattern to the correct solution. Recall from Figure 1, the holding fix is located at the point $\bar{x}=0, \bar{y}=0$. Since the aircraft starts at the holding fix and then flies four segments, returning back to the holding fix, we can write the following two equations for $\bar{x}$ and $\bar{y}$

$$\bar{x} = (\text{Cos}\,\theta_H - \bar{V}_W\,\text{Cos}\,\alpha)t_{out} + (\text{Cos}\,\sigma - \bar{V}_W\,\text{Cos}\,\alpha)(60\beta) - (\frac{360}{k})\bar{V}_W\,\text{Cos}\,\alpha \qquad (83)$$

$$\bar{y} = (\text{Sin}\,\theta_H - \bar{V}_W\,\text{Sin}\,\alpha)t_{out} - (\frac{360}{k})\bar{V}_W\,\text{Sin}\,\alpha \qquad (84)$$

Note that when k=3 degrees/sec, the last term in the above equations represents the two- minute time to turn 360 degrees, multiplied by the component of the wind in the x and y directions. The second term in eq. (83) corresponds to inbound leg of the holding fix. The terms which have t_{out} as multipliers, represent the distance traveled in the x and y directions during the outbound time. For a given windspeed and direction, any error in the aircraft position when returning to the holding fix, must be corrected on the outbound leg. Thus, the deviation from $\bar{x}=0, \bar{y}=0$, can be obtained by differentiating equations (83) and (84) with respect to the variables t_{out} and θ_H, i.e.

$$
\begin{aligned}
d\bar{x} &= (\frac{\partial \bar{x}}{\partial t_{out}})\Delta t_{out} + (\frac{\partial \bar{x}}{\partial \theta_H})\Delta \theta_H \\
d\bar{y} &= (\frac{\partial \bar{y}}{\partial t_{out}})\Delta t_{out} + (\frac{\partial \bar{y}}{\partial \theta_H})\Delta \theta_H
\end{aligned}
\qquad (85)
$$

The partial derivatives multiplying the terms Δt_{out} and $\Delta \theta_H$ are called influence coefficients, since they relate changes in $d\bar{x}$, which is related to an incorrect inbound time, and $d\bar{y}$, which is related to an undershoot or overshoot in re-intercepting the inbound course. The above equations introduce an important concept in the holding pattern that is usually never discussed during IFR training. The concept is called the "Coupling Effect". The "Coupling Effect" brings in the important concept that changes needed to the inbound time can occur by changes in both outbound time and OWCA. Whereas, changes needed to correct undershoots/overshoots can be made by changes in both outbound time and OWCA. This is in contrary to the IFR training methods stated above, that changes in inbound time are corrected by a change in outbound time, and changes in undershoots/overshoots are corrected by a change in OWCA.

As an example, consider the case $\bar{V}_W = 0.3$ and $\alpha = 45$ degrees, k=3, and $\beta = 1$. Figures 12 and 13 demonstrate the concept of the "Coupling Effect". In Figure 12, we show that when the outbound heading is held constant, changing the outbound time modifies both the inbound time and the overshoot/undershoot of the inbound course. Similarly, Figure 13 shows that when the outbound time is held constant, changes in OWCA modify both the inbound time and the overshoot/undershoot of the inbound course.

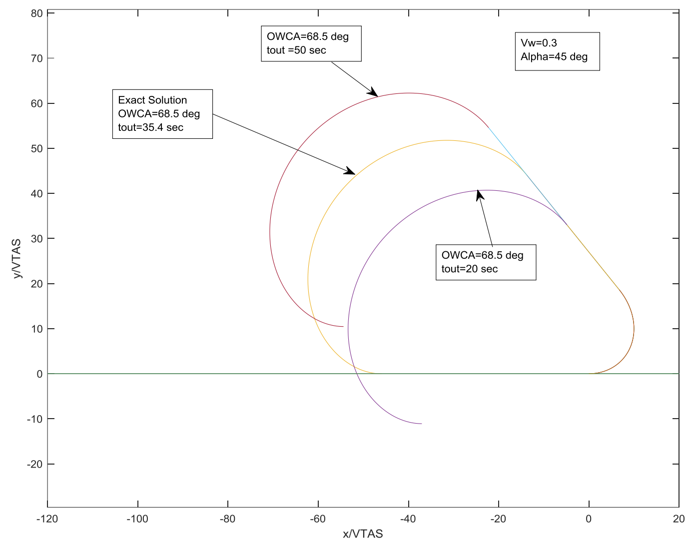


Figure 12: Effect of Outbound Time on Inbound Course Undershoot/Overshoot

(OWCA Held Constant)

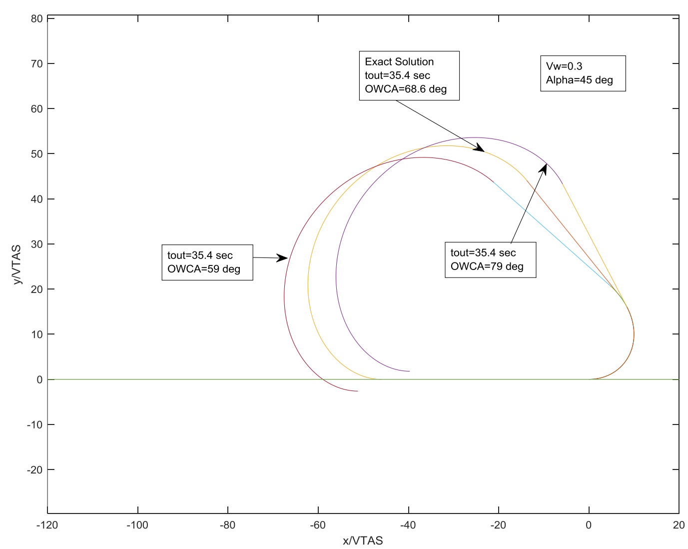


Figure13: Effect of OWCA on Inbound Time (t_{out} Held Constant)

The influence coefficients can be calculated directly from eqs. (85), i.e.

$$\frac{\partial \overline{x}}{\partial t_{out}} = \text{Cos}\,\theta_H - \overline{V}_W\,\text{Cos}\,\alpha$$
$$\frac{\partial \overline{x}}{\partial \theta_H} = -t_{out}\,\text{Sin}\,\theta_H \qquad (86)$$
$$\frac{\partial \overline{y}}{\partial t_{out}} = \text{Sin}\,\theta_H - \overline{V}_W\,\text{Sin}\,\alpha$$
$$\frac{\partial \overline{y}}{\partial \theta_H} = t_{out}\,\text{Cos}\,\theta_H$$

Since we are after the required changes in both Δt_{out} and $\Delta\theta_H$, we need to invert eq. (85) to obtain these variables in terms of the observed changes in both $d\overline{x}$ and $d\overline{y}$. Performing this inversion gives us the following equations for both Δt_{out} and $\Delta\theta_H$

$$\Delta t_{out} = \frac{\text{Cos}\,\theta_H d\overline{x} + \text{Sin}\,\theta_H d\overline{y}}{[1 - \overline{V}_W\,\text{Cos}(\theta_H - \alpha)]} \qquad (87)$$

and

$$\Delta\theta_H = \frac{[\text{Cos}\,\theta_H - \overline{V}_W\,\text{Cos}\,\alpha]d\overline{y} - [\text{Sin}\,\theta_H - \overline{V}_W\,\text{Sin}\,\alpha]d\overline{x}}{t_{out}[1 - \overline{V}_W\,\text{Cos}(\theta_H - \alpha)]} \qquad (88)$$

We can also relate the quantity $d\overline{x}$ to the error in the inbound time using the following equation

$$d\overline{x} = [\text{Cos}\,\sigma - \overline{V}_W\,\text{Cos}\,\alpha](t_{in} - 60\beta) \qquad (89)$$

Note that the term in brackets is just the aircraft groundspeed on the inbound course to the fix, multiplied by the difference between the inbound time and the required inbound time. If the inbound time is less than the required inbound time, the aircraft would be beyond the holding fix after a time of 60β, and thus, the Pilot must introduce a value of $d\overline{x} < 0$ in order to bring the aircraft back to the holding fix. When the inbound time is greater than 60β, $d\overline{x} > 0$ in order to bring the aircraft forward to the holding fix. Thus, early arrival requires $d\overline{x} < 0$, and late arrival requires $d\overline{x} > 0$. In regard to the aircraft overshooting or undershooting the inbound course, the following is true: (a) Overshooting the inbound course places the aircraft at a value of $\overline{y} < 0$, and thus, to bring the aircraft upward onto the inbound course requires a value of $d\overline{y} > 0$, and (b) Similarly, undershooting the inbound course requires a value of $d\overline{y} < 0$ in order to move the aircraft downward onto the inbound course. Thus, after each complete circuit back to the holding fix, we obtain the estimated values of $d\overline{x}$ and $d\overline{y}$ necessary to bring the aircraft back to the holding fix. Using these values and the values of θ_H and t_{out} used in the just completed circuit, we can calculate the required changes in both t_{out} and θ_H for the next circuit. This convergence process is shown in Figure 14. Here we see that the initial holding pattern (circuit 1), as recommended by the AIM (i.e. an M-factor=3) is quite far off from the exact solution of the holding pattern under these wind conditions. The outbound time and heading for circuit 2 are obtained by adding the changes in eqs. (87) and (88) to the values of t_{out} and θ_H used in the previous circuit (i.e. circuit 1). This process is then used for each subsequent circuit, until the holding pattern converges to the correct solution. Note that using this "Smart-Convergence" algorithm, the holding pattern is essentially converged in two additional circuits, i.e. by circuit 3. We have also shown the exact solution for this holding pattern for comparison purposes. It is important to point out that even with the "Smart-Convergence" algorithm, the initial guess for the holding pattern is extremely important in rapidly converging to the exact solution. For example, if circuit 2 was the initial guess, the holding pattern would converge by the next circuit using this "Smart-Convergence algorithm. Table 1 shows the results of the actual track of the holding pattern during this convergence process. In Table 1, we have overlaid the aircraft Heading Indicator, so that we show the actual outbound heading for these circuits. Clearly, by circuit 3 we have come close to convergence, and by circuit 4, we have converged to the exact solution.

We should also point out that there are two interesting cases that arise when analyzing eqs. (87) and (88). In eq. (87), we see that the numerator becomes zero when

$$\text{Cos}\,\theta_H d\bar{x} + \text{Sin}\,\theta_H d\bar{y} = 0 \qquad (90)$$

Under this condition, there would be no required change in Δt_{out}. Equation (90) can be rewritten as the dot product of the normalized TAS vector along the outbound segment, i.e. $\bar{V} = \text{Cos}\,\theta_H \hat{i} + \text{Sin}\,\theta_H \hat{j}$ and $\varepsilon = d\bar{x}\hat{i} + d\bar{y}\hat{j}$. Where $\hat{i}$ and $\hat{j}$ are the unit vectors in the x and y directions, and ε is the error vector between the holding fix and where the aircraft is located at the end of the required inbound time, 60β. Equation (90) is satisfied when the TAS vector on the outbound segment is orthogonal to the error vector for the just completed circuit. In this case, any changes in t_{out} will not change the error vector.

In a similar fashion, the numerator of eq. (88) is identically zero when

$$\frac{d\bar{y}}{d\bar{x}} = \frac{[\text{Sin}\,\theta_H - \bar{V}_W\,\text{Sin}\,\alpha]}{[\text{Cos}\,\theta_H - \bar{V}_W\,\text{Cos}\,\alpha]} \quad (91)$$

Equation (91) shows that when the groundspeed vector on the outbound segment is parallel to the error vector in the just completed circuit, there will be no required change in the outbound heading for the next circuit. A change in outbound time will just change the magnitude of the error vector and not its direction. For example, if the correct value of t_{out} was used on the next circuit, the error vector would be identically zero upon returning to the holding fix.

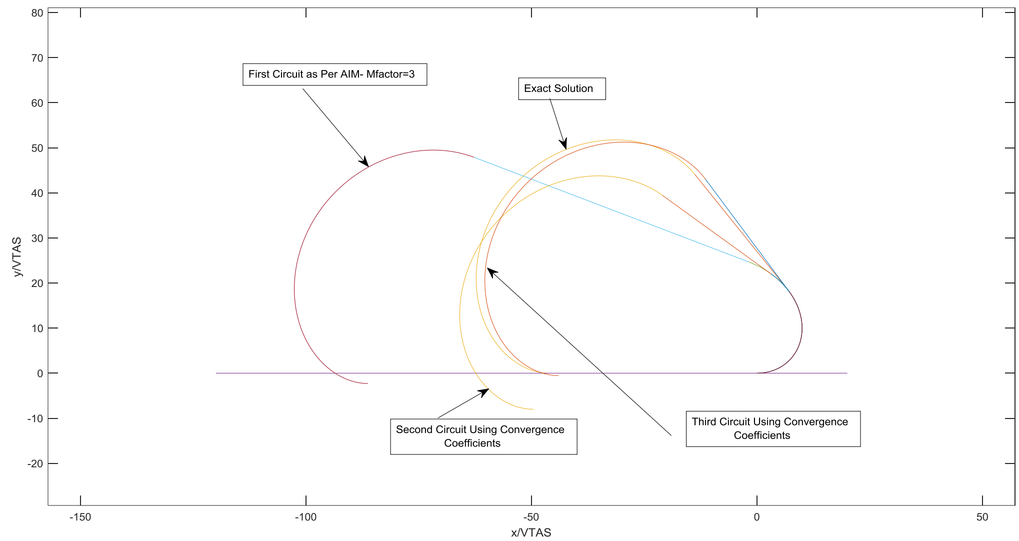


Figure 14: Holding Pattern Convergence Using Smart-Convergence Algorithm

Table 1: Using the Holding Pattern Convergence Coefficients ($\bar{V}_W = 0.3$, $\alpha = 45$ degrees)

Circuit Number	Time Out (sec)	Heading Out (sec)	Time In (sec)	Centerline Error/VTAS (sec)
1	60	216.7	112.7	-2.3
2	30.3	233.1	65.4	-7.6
3	34.1	250.3	57.7	-0.6
4	35.4	248.5	60.0	-0.03

We now consider an experienced IFR Pilot attempting to converge the holding pattern using the “Bracketing Method” they were taught during their IFR training. The initial holding pattern is identical to that shown in Figure 14. Table 2 shows the corrections the IFR Pilot is making while attempting to fly the holding pattern. Figure 15 shows the actual track of the aircraft during the IFR Pilot’s attempt to converge to the exact solution of the holding pattern using the “Bracketing Method”. Note that by the end of circuit 5 the IFR Pilot, although close, has still not converged to the correct holding pattern. In fact, the error in both the outbound time and outbound heading after 5 circuits is nearly identical to the “Smart-Convergence” algorithm after 3 circuits. This difference in the number of circuits is due to the “Coupling Effect” not being taken into account during the IFR Pilot’s attempt to converge to the correct holding pattern.

Table 2: Average IFR Pilot Attempting to Converge to the “Holding Pattern” Solution

($\bar{V}_W = 0.3, \alpha = 45$ degrees)

Circuit Number	Time Out (seconds)	Heading Out (degrees)	Time In (seconds)	Centerline Error/VTAS (seconds)
1	60	217	113	-2.3
2	30	232	66	-8.2
3	27	257	49	-4.9
4	33	247	59	-2.1
5	35	251	58	0.2

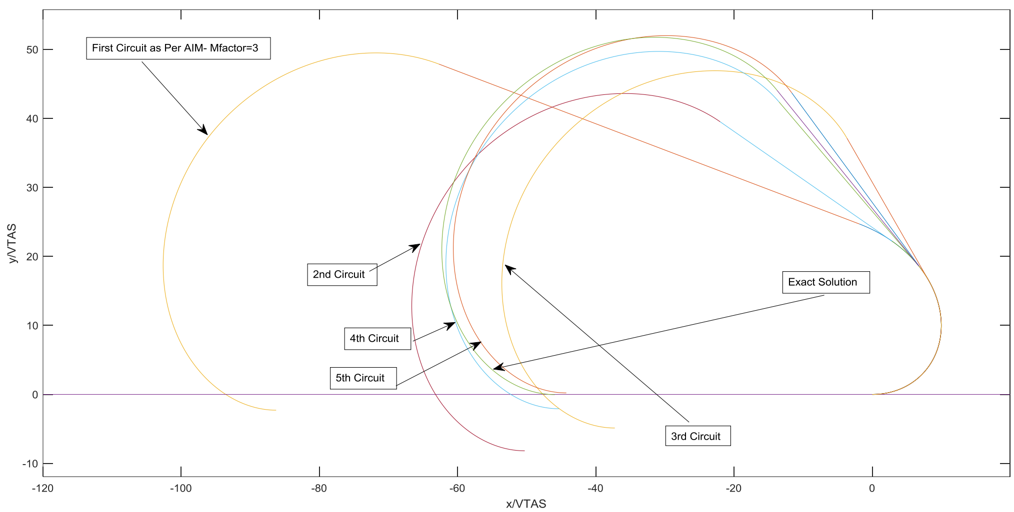


Figure 15: Standard “Bracketing Method” Used to Converge the Holding Pattern

It is clear that there are two causes requiring the IFR Pilot to need additional circuits to converge to the correct holding pattern solution. The first being the poor initial circuit that is based on the AIM recommendations, and the second being the obvious lack of understanding of the “Coupling Effect” between the outbound time and the outbound heading and their relationship to the incorrect inbound time and the overshoots/undershoots that occur while attempting to re-intercept the inbound course.

In Section 4 we develop a form of the “Smart-Convergence” algorithm that can be used in the “Bracketing Method” that will allow one to converge the holding pattern using fewer circuits.

4.0 Detailed Comparison of the "Bracketing Method" and the "Smart - Convergence" Algorithm for Type-1 Holding Patterns

Clearly, the simplest method of nailing the holding pattern down correctly is to pick the outbound time and the M-Factor off Figures 6 and 7a. In fact, even eyeballing the inbound time and M-Factor would most probably nail the holding pattern down in two circuits. One might challenge the method by stating that we do not know the wind accurately. However, using the latest technology of the GPS (or how about the E6B?), should be able to get a reasonably accurate current wind speed and direction, provided the Pilot inputs the correct TAS and WCA (i.e. solving the inverse "Wind Triangle Problem").

As we all know, doing mental arithmetic in the cockpit during a hold is a challenging task in itself, even for a Mathematician. However, we can develop a Table which tells the Pilot which direction to make changes in the outbound time and outbound heading, based on the inbound time and the overshoot/undershoot that occurs during the time the aircraft is re-intercepting the inbound course. In order to develop this Table, we will use the "Smart-Convergence" algorithm discussed in Section 3. This Table is a Tick-Tack-Toe Board with the three columns representing undershooting, on course, and overshooting the inbound course. Whereas, the three rows represent the aircraft arriving at the holding fix before the required inbound time, on time, or arriving at the holding fix later than the required inbound time. Note that the center box corresponds to the aircraft being on time and on the inbound course, and therefore, no changes are required.

We first start by determining the sign of the required changes in the quantities $d\bar{x}$ and $d\bar{y}$, and the product of $d\bar{x} * d\bar{y}$, in each of the nine boxes on the Tick-Tack-Toe Board, i.e.

Table 3: Sign and product of $d\bar{x}$ and $d\bar{y}$ in Converging to Holding Pattern Solution

	Undershoot (dy<0)	On Course (dy=0)	Overshoot (dy>0)
Inbound Time Early (dx<0)	dx<0 dy<0 dx*dy>0	dx<0, dy=0 dx*dy=0	dx<0 dy>0 dx*dy<0
On Time (dx=0)	dx=0 dy<0 dx*dy=0	dx=0 dy=0 dx*dy=0 Converged	dx=0 dy>0 dx*dy=0
Inbound Time Late (dx>0)	dx>0 dy<0 dx*dy<0	dx>0 dy=0 dx*dy=0	dx>0 dy>0 dx*dy>0

Recall that the center box is the converged holding pattern solution. In this case, the Pilot should continue to fly the previous outbound time and OWCA. Since the wind has some variability over 5-10 minutes, the holding pattern may change slightly from one circuit to the next. However, the perturbation should be minor, and in general, the solution should be close to the target solution previously achieved.

Equations (87) and (88) can now be used to determine whether $\Delta t_{out} > 0$ or $\Delta t_{out} < 0$, and whether $\Delta\theta_H > 0$ or $\Delta\theta_H < 0$. Note that the denominator of the coefficients in eqs. (87) and (88) is always greater than zero. Since we are addressing Type-1 holding patterns, where the relative outbound heading is somewhere between 90 and 180 degrees, we know that the sign of the coefficients of $d\bar{x}$ and $d\bar{y}$ are bounded in the following manner

$$\begin{aligned} &\mathrm{Cos}\,\theta_H < 0 \\ &\mathrm{Sin}\,\theta_H > 0 \end{aligned} \qquad (92)$$

The coefficient of $d\bar{x}$ in eq.(88) can be rewritten as $\mathrm{Sin}\,\sigma[\frac{\mathrm{Sin}\,\theta_H}{\mathrm{Sin}\,\sigma} - 1]$. Using eqs. (47), (49) and (53), we see that $[\frac{\mathrm{Sin}\,\theta_H}{\mathrm{Sin}\,\sigma} - 1]$ is greater than zero for $\bar{V}_W < \frac{1}{3}$. Manipulating eq. (22), and using the fact that $[\frac{\mathrm{Sin}\,\theta_H}{\mathrm{Sin}\,\sigma} - 1] > 0$, we see that $\mathrm{Cos}\,\theta_H - \bar{V}_W\,\mathrm{Cos}\,\alpha < 0$ for $\bar{V}_W < \frac{1}{3}$. Therefore, the signs of the remaining two coefficients are shown to be

$$\begin{aligned} &\mathrm{Sin}\,\theta_H - \bar{V}_W\,\mathrm{Sin}\,\alpha > 0 \\ &\mathrm{Cos}\,\theta_H - \bar{V}_W\,\mathrm{Cos}\,\alpha < 0 \end{aligned} \qquad (93)$$

We will first concentrate on the required changes in the outbound time for required changes in both $d\bar{x}$ and $d\bar{y}$. The Tick-Tack-Toe Board can now be filled in with the required Δt_{out}. This is shown below in Table 4.

Table 4: Required Changes in Outbound Time t_{out}

	Undershoot (dy<0)	On Course (dy=0)	Overshoot (dy>0)
Inbound Time Early (dx<0)	dx<0 dy<0 $\Delta t_{out} = ?$	dx<0, dy=0 $\Delta t_{out} > 0$	dx<0 dy>0 $\Delta t_{out} > 0$
On Time (dx=0)	dx=0 dy<0 $\Delta t_{out} < 0$	dx=0 dy=0 Converged	dx=0 dy>0 $\Delta t_{out} > 0$
Inbound Time Late (dx>0)	dx>0 dy<0 $\Delta t_{out} < 0$	dx>0 dy=0 $\Delta t_{out} < 0$	dx>0 dy>0 $\Delta t_{out} = ?$

Note that we observe that upper left and lower right corner boxes show $\Delta t_{out} = ?$. The "Smart-Convergence" algorithm shows that in these two cases, the $d\overline{x}$ and $d\overline{y}$ contributions to Δt_{out} are of opposite sign, so that one needs to know the magnitude of each term before we can determine whether to increase or decrease the outbound time.

In a similar fashion, we can generate the equivalent table for the outbound heading θ_H. This is shown below in Table 5 below.

Table 5: Required Changes in the Outbound Heading θ_H

	Undershoot (dy<0)	On Course (dy=0)	Overshoot (dy>0)
Inbound Time Early (dx<0)	dx<0 dy<0 $\Delta\theta_H > 0$	dx<0, dy=0 $\Delta\theta_H > 0$	dx<0 dy>0 $\Delta\theta_H = ?$
On Time (dx=0)	dx=0 dy<0 $\Delta\theta_H > 0$	dx=0 dy=0 Converged	dx=0 dy>0 $\Delta\theta_{out} < 0$
Inbound Time Late (dx>0)	dx>0 dy<0 $\Delta\theta_H = ?$	dx>0 dy=0 $\Delta\theta_H < 0$	dx>0 dy>0 $\Delta\theta_H < 0$

In this case, the upper right and lower left corner boxes in Table 5 show $\Delta\theta_H = ?$ Again, this indicates that the contributions from $d\overline{x}$ and $d\overline{y}$ are of opposite signs, and thus, one needs to know the magnitude of each term before we can determine whether to increase or decrease the outbound heading.

Since θ_H is the outbound heading relative to the inbound course, it is best to describe the required changes in terms of the outbound time and the OWCA. Recall that increasing θ_H reduces the OWCA and decreasing θ_H increases the OWCA. Equation (68) shows the relationship between the OWCA (δ) and the relative outbound heading θ_H. Table 6 shown below, is now described in terms of required changes in outbound time and OWCA.

Table 6: Pilot Corrective Actions to Converge to the Holding Pattern

	Undershoot (dy<0)	On Course (dy=0)	Overshoot (dy>0)
Inbound Time Early (dx<0)	Change in t_{out}=? Decrease OWCA	Increase t_{out} Decrease OWCA	Increase t_{out} Change in OWCA=?
On Time (dx=0)	Decrease t_{out} Decrease OWCA	No Change in t_{out} and OWCA	Increase t_{out} Increase OWCA
Inbound Time Late (dx>0)	Decrease t_{out} Change in OWCA=?	Decrease t_{out} Increase OWCA	Change in $t_{out=?}$ Increase OWCA

Note that the four corners of Table 6, where both $d\overline{x}$ and $d\overline{y}$ are non-zero will always have some uncertainty in either the outbound time or the OWCA, due to the competing effects of the $d\overline{x}$ and $d\overline{y}$ terms described earlier.

We now compare the results of Table 6, which utilizes the information from the "Smart-Convergence" algorithm, with the "Bracketing Method" taught to IFR Pilots during their training on holding patterns. In order to make the comparison, we highlight in green all changes that agree between the "Bracketing Method" and the "Smart-Convergence" algorithm. We highlight in red, all changes that are not consistent between the two methods. In addition, we highlight in Cyan, changes that are required by the "Smart-Convergence" algorithm, but not required in the "Bracketing Method". These results are shown in Table 7 below.

Table 7: Comparison of Corrective Actions between Bracketing Method and Smart-Convergence Tool Algorithm

	Undershoot (dy<0)	On Course (dy=0)	Overshoot (dy>0)
Inbound Time Early (dx<0)	Increase t_{out} Decrease OWCA	Increase t_{out} Decrease OWCA	Increase t_{out} Increase OWCA
On Time (dx=0)	Decrease t_{out} Decrease OWCA	No Change in both t_{out} and OWCA	Increase t_{out} Increase OWCA
Inbound Time Late (dx>0)	Decrease t_{out} Decrease OWCA	Decrease t_{out} Increase OWCA	Decrease t_{out} Increase OWCA

It is clear from Table 7 that other than the center box, which corresponds to having achieved the correct holding pattern, all boxes have only one correct Pilot response (as highlighted in green). The corner boxes in Table 7 show four responses in red which may or may not be the correct response. This is due to the competing terms attributed to the "Coupling Effect". In addition, the boxes which have either $d\bar{x}$ or $d\bar{y}$ equal to zero, only have one input change when using the "Bracketing Method", whereas, the "Smart-Convergence" algorithm requires an additional change in the other variable (i.e. highlighted in Cyan), due to the "Coupling-Effect".

Based on the above Tables, we identify the reason it takes more circuits to converge to the "Holding Pattern" solution when using the "Bracketing Method". It boils down to the lack of accounting for the "Coupling Effect" during the convergence process. Table 7 truly shows the complexity of the convergence process when the winds are not light, as is demonstrated in this particular example. Therefore, if CFI-I's are going to continue to train IFR Pilots in using the "Bracketing Method", it is imperative to have the first circuit as close as possible to the converged holding pattern, in order to avoid spending a considerable amount of time trying to converge to the "Holding Pattern" solution. If the initial circuit is far off, one may ask the question, is there a way of obtaining a better update for the next circuit without using the "Smart -Convergence" algorithm?

As an example, in the case of $\bar{V}_W = 0.3$ and $\alpha = 45$ degrees, we note that Table 2 showed that using an M-Factor of 3 with a 60 second outbound time, took 113 seconds to return to the holding fix, while at the same time, overshooting the inbound course. In this case, Table 7 indicates that we need to increase the OWCA (i.e. highlighted in green) and decrease the outbound time (i.e. highlighted in red). The "Smart-Correction" algorithm shows the required correction in $d\overline{x}$ is considerably larger than the required correction in $d\overline{y}$,and thus, the outbound time should decrease. This is consistent with the decrease in outbound time highlighted in red in Table 7. Recall that Table 7 only provides the direction of the change in both OWCA and outbound time, and not the magnitude of the change. However, let us consider using eq. (42) in an attempt to predict a best guess for the required magnitude of the change in the outbound time. In the first circuit, the aircraft flew a relative outbound heading of 143 degrees, with the IWCA=12 degrees. Substituting the required values into eq. (42), gives the following relationship between the required change in the outbound time and the required changed in the inbound time i.e. $\Delta t_{out} = 0.76\Delta t_{in}$. Therefore, if we hold the OWCA constant during the next circuit, we would need to decrease the outbound time by 0.76*53=40 seconds. Thus, the next outbound time would be 20 seconds while holding the relative outbound heading at 143 degrees. This is shown in Figure 16 below. It is clear that using a 20 second outbound time on circuit 2 gives rise to exactly 60 seconds for the inbound time, although the “Coupling Effect” has caused the aircraft to further overshoot the inbound course without the needed increase in the OWCA. However, this does confirm the use of eq. (42) to obtain a first guess for the corrected outbound time for the second circuit.

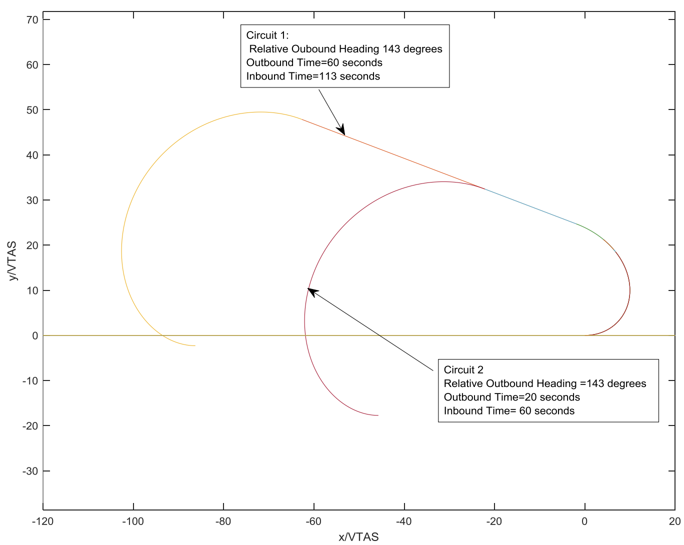


Figure 16: Estimating Change in Outbound Time for Required Change in Inbound Time

It is important to point out that although eq. (42) gave an accurate first guess for the second circuit outbound time, the IFR Pilot still cannot perform the mental arithmetic required while flying the holding pattern in IFR conditions. It appears that the best option for the IFR Pilot to be able to converge to the “Holding Pattern” solution in a minimum number of circuits, would be to eyeball Figures 6 and 7a to obtain the outbound time and M-factor for the prescribed wind speed and direction. Although eyeballing both Figures 6 and 7a should not be a problem, let us consider the Pilot just eyeballing the outbound time from Figure 7a, and using the AIM recommendation for the M-Factor as 3. If we eyeball Figure 7a for the case $\bar{V}_W = 0.3$ and $\alpha = 45$ degrees, we find the outbound time to be approximately 35 seconds. Figure 17 below, shows both the exact solution for this holding pattern, as well as the track of three consecutive circuits, during which the outbound time was held constant at 35 seconds, while the IFR Pilot converged on the required OWCA.

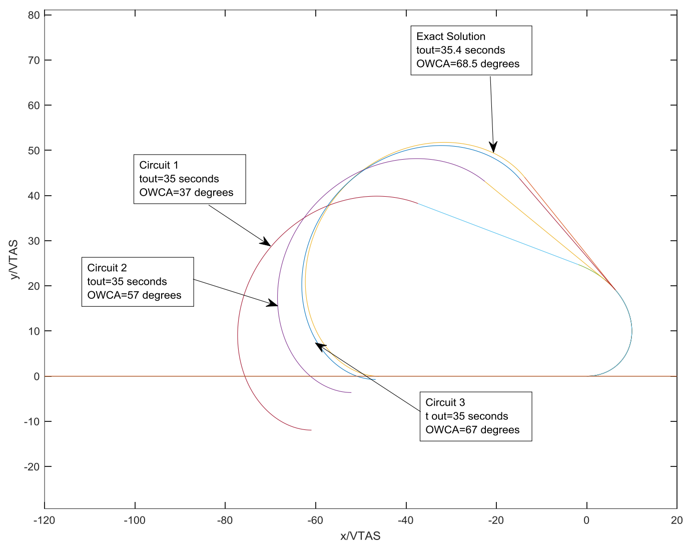


Figure 17: Eyeballing Outbound Time from Figure 7a

Note that by eyeballing the outbound time from Figure 7a, the convergence to the exact solution of the holding pattern occurs in three circuits, compared to the previous convergence process, where the Pilot was converging on both the outbound time and OWCA. Table 8 below, shows the details of the convergence process for these three circuits.

Table 8: Convergence History of Holding Pattern by Eyeballing tout from Figure 7a

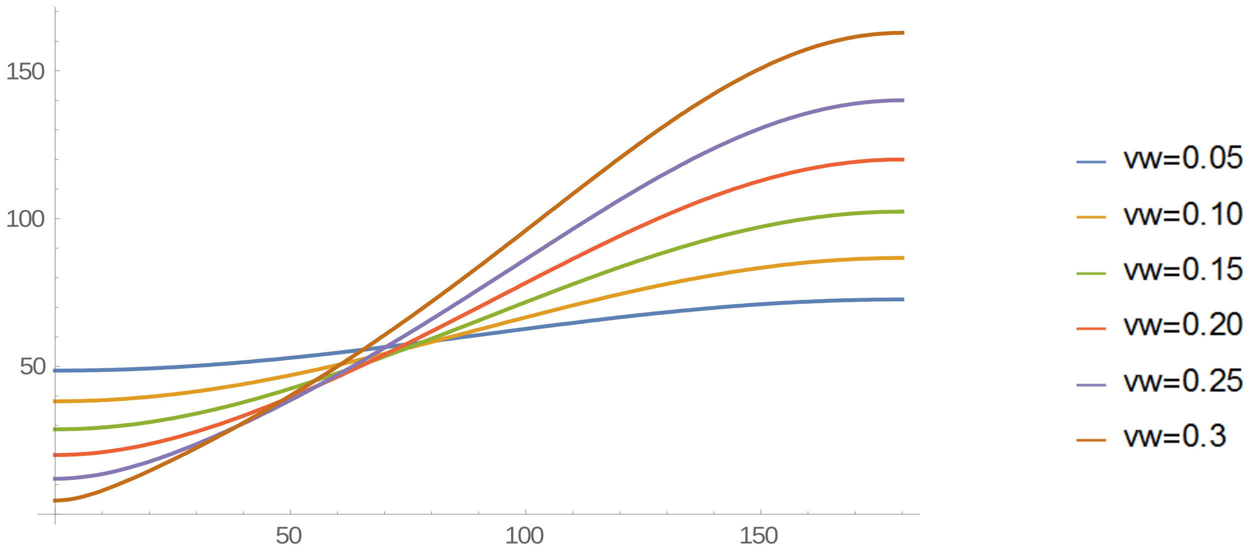
Circuit Number	Outbound Time (seconds)	Outbound Heading (degrees)	Inbound Time (seconds)	Centerline Error/VTAS (seconds)
1	35	143.3	79.6	-11.9
2	35	123.3	68.1	-3.6
3	35	113.3	61.1	-0.7

Here we see that at the end of the first circuit, the aircraft has overshot the inbound course by a considerable amount (i.e. at V_{TAS}=90 knots, by nearly 1800 feet). The corresponding inbound time is 79.6 seconds. This locates the aircraft in the lower right corner box in Table 7. Since we are not changing the outbound time, we need to increase the OWCA. As a first correction to the OWCA we increase the OWCA by 20 degrees, flying a relative outbound heading of 123.3 degrees. At the completion of the second circuit, the aircraft has overshot the inbound course again, however only by about 545 feet. The inbound time is now 68.1 seconds. Again, this locates the aircraft in the lower right corner box, which requires an additional increase in the OWCA. However, this time we increase the OWCA by only 10 degrees. The aircraft relative outbound heading is now 113.3 degrees. At the completion of the third circuit, the aircraft overshoots the inbound course by approximately 105 feet. The inbound time for this circuit is now 61.1 seconds. For all practical purposes, the Pilot has converged to the holding pattern solution in three circuits, rather than the five circuits previously required when the Pilot was converging on both the outbound time and OWCA.

The above results indicate that the IFR Pilot should at least have Figure 7a handy, since eyeballing the outbound time can cut the number of required circuits by about 40% in order to converge to the “Holding Pattern” solution. However, having Figure 6 handy also, would allow the Pilot to converge to the “Holding Pattern” solution by the second circuit. Figure 18 shows both Figures 6b and 7a on one page that can easily be eyeballed for both the outbound time and M-Factor.

(a) Outbound Time: t_{out} (seconds) Measured from Outbound Heading

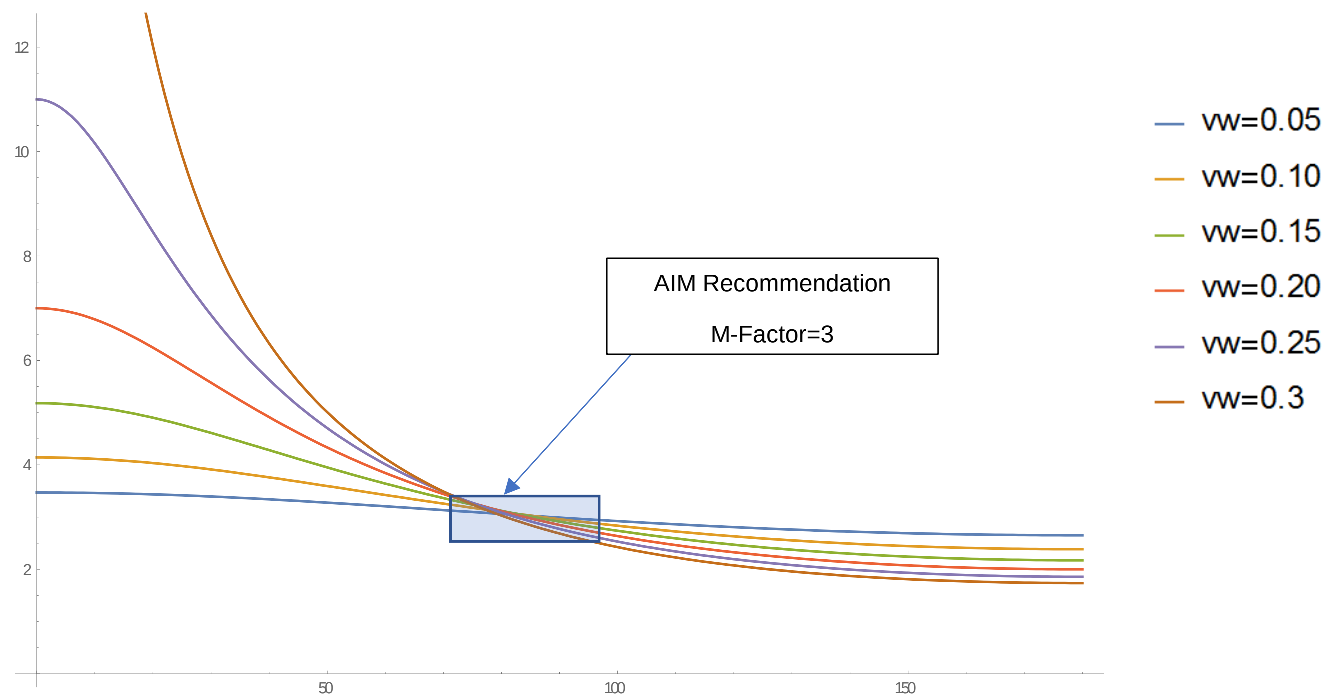
t_{out} (sec)



Relative Wind Angle α (degrees)

(b) M-Factor$=\dfrac{OWCA}{IWCA}$

M-Factor



Relative Wind Angle α (degrees)

Figure 18: Holding Pattern Solution: (a) Outbound Time, and (b) M-Factor

5.0 Preparing for the Hold

In preparation for the hold, the IFR Pilot should ensure: (a) The Heading Indicator is set accurately, and (b) The outer scale of the airspeed indicator reads the correct TAS for the pressure altitude and the OAT. This is necessary in order to obtain a reasonably accurate solution for the windspeed and wind direction. In today's use of GPS on many GA Aircraft, the Pilot is able to display the wind speed and wind direction on the GPS display. For example, on the Garmin 400W, the AUX page denoted as "Density Alt/TAS/Winds" can be selected. This page requires the following information to be entered into the GPS: (1) Indicated altitude, (2) CAS, (3) Barometric pressure, (4) TAT (total air temperature), and (5) Aircraft heading. The GPS output corresponding to above input parameters are: (a) Density altitude, (b) TAS, (c) Wind direction, (d) Wind speed, and (e) Headwind or tailwind component. The methodology the GPS uses to obtain the wind direction and windspeed is calculated using eqs. (1) and (2) shown below.

$$\begin{aligned} &\mathrm{Sin}\,\sigma = \bar{V}_W\,\mathrm{Sin}\,\alpha \\ &\bar{V}_G = \frac{V_G}{V_{TAS}} = \mathrm{Cos}\,\sigma - \bar{V}_W\,\mathrm{Cos}\,\alpha \end{aligned} \quad (94)$$

Since the GPS knows both the TAS and groundspeed, the left-hand side of the second equation in eq. (94) is known. When tracking directly to the holding fix, knowing the aircraft heading and course to the fix, the WCA σ can be determined. Thus, the groundspeed equation can be rearranged to give

$$\bar{V}_W\,\mathrm{Cos}\,\alpha = \mathrm{Cos}\,\sigma - \bar{V}_G \quad (95)$$

Note the first equation in eq. (94) is just

$$\bar{V}_W\,\mathrm{Sin}\,\alpha = \mathrm{Sin}\,\sigma \quad (96)$$

Dividing eq. (96) by eq. (95) gives the following result for the wind angle

$$\frac{\mathrm{Sin}\,\alpha}{\mathrm{Cos}\,\alpha} = \mathrm{Tan}\,\alpha = \frac{\mathrm{Sin}\,\sigma}{\mathrm{Cos}\,\sigma - \bar{V}_G} \quad (97)$$

Taking the inverse tangent of both sides of eq. (97) gives the final result for the wind direction relative to the course the course being tracked, i.e.

$$\alpha = \mathrm{Tan}^{-1}\left[\frac{\mathrm{Sin}\,\sigma}{\mathrm{Cos}\,\sigma - \bar{V}_G}\right] \quad (98)$$

Here α is in radians, and to convert to degrees we multiply the resultant value of α by $180/\pi$. Finally, with the relative wind angle known, either eq. (95) or eq. (96) can be used to obtain the value of $\bar{V}_W$, and thus the windspeed. Once the value of alpha and the aircraft heading are specified, the wind direction can be determined.

At this point, one might ask the question, “Why not program eq. (15)) for the outbound time and eq. (22) for the outbound heading directly into the GPS”. Since the GPS already knows the windspeed and direction, the only additional information needed is the inbound course to the holding fix. The GPS can then predict the correct outbound time and outbound heading for the holding pattern. Since the GPS is constantly updating the windspeed and direction, the GPS should be able to continually update the outbound time and outbound heading for the next circuit. This approach would reduce the workload of the IFR Pilot in attempting to converge to the “Holding Pattern” solution. It is not clear as to why the manufacturers have not implemented this into their GPS software, unless they are not aware of the fact that an exact solution to the “Holding Pattern” problem does exist.

6.0 Recommendations for IFR Training on Holding Patterns

The fact that the “Holding Pattern” solution has been derived and is in the open literature as of this writing, it is important that CFI-I’s take advantage of the knowledge provided in this Treatise. For example, it is important for the IFR Pilot to understand that without the GPS providing the “Holding Pattern” solution, the “Bracketing Method” will have to be utilized. Therefore, it is important for all CFI-I’s to convey the following information to their instrument Students:

(1) CFI-I’s should introduce their Students to the Type-2 holding pattern. This can be performed either on a Simulator, or in the air by reducing the TAS to the point where the windspeed ratio is above $\frac{1}{3}$. Having the IFR Student try to fly this holding pattern will provide the knowledge that these types of holding patterns should be avoided at all cost due to the lack of a systematic convergence process. CFI-I’s should teach their IFR Students to fly the holding pattern with the windspeed ratio kept at or below 0.25 in order to provide enough outbound time before turning back to re-intercept the inbound course.

(2) CFI-I’s training IFR Students on Type-1 holding patterns should emphasize the fact that the M-Factor is not always close to 3 and can range from just below two to considerably greater than 3, depending on the windspeed ratio and direction. In order to demonstrate this large variation in the M-Factor, it is important to fly the holding pattern with the following wind directions

a. Direct headwind (α =0)
b. α =45
c. α =90

d. $\alpha = 135$
e. Direct tailwind (α =180)

If this training is performed in the air, changing the inbound course in the holding pattern can provide the required values of α in (a) – (e).

(3) Fly the holding pattern with the wind direction from $\pm\alpha$ to demonstrate that the outbound time will be same if started from the point at which aircraft has turn to the outbound heading but will be different if started at the abeam point. Although the abeam point is given priority in the AIM for starting the outbound time, it is the inbound time that needs to be satisfied while remaining in the holding pattern protected airspace.

(4) Vary the TAS for a given windspeed in order to show the importance of the windspeed ratio as one of the key parameters that control the shape of the holding pattern.

(5) Introduce the use of Table 7 and the concept of the "Coupling Effect" when converging to the "Holding Pattern" solution. Provide Figure 18 to the IFR Student as an aid in reducing the number of circuits necessary to converge to the "Holding Pattern" solution.

7.0 Conclusions

In this Treatise we have derived the exact solution of the "Holding Pattern" problem. The exact solution provides us with the following information: (a) The IWCA, (b) The outbound heading (or the OWCA), and (c) The outbound time measured from the time the aircraft completes the turn to the outbound heading, all as a function of the windspeed ratio ($\bar{V}_W$) and the wind angle relative to the inbound course (α). The exact solution contains two algebraic equations for the outbound heading and the outbound time.

The exact solution provides a number of interesting observations that has never been previously discussed in the open literature:

(1) Specifically, there are advantages to starting the outbound time when the aircraft completes the turn to the outbound heading, rather than at the abeam point. These advantages are: (a) For a given value of α, the outbound time is the same, independent of whether the wind is coming from the holding or non-holding side, and (b) The Pilot does not need to determine the abeam point. Point (a) is important because one only needs to determine the solution for $0 \le \alpha \le 180$, rather than for $0 \le \alpha \le 360$. In addition, the OWCA for $\alpha \le 0$ is just the negative of the OWCA for $\alpha \ge 0$.

(2) Type-2 holding pattern: When the windspeed ratio is greater than one-third while holding with a headwind component, it is impossible to make the inbound time one minute unless the aircraft reaches the holding fix and turns to an outbound heading which is less than 90 degrees to the inbound course. When the relative heading for the outbound turn is between 45 and 90 degrees from the inbound course, the outbound time controls the overshoots/undershoots, whereas the OWCA controls the inbound time. This is contrary to way we normally train IFR pilots. As a consequence, in order to avoid flying a Type-2 holding patterns it is recommended that all IFR Pilot's ensure that they fly the holding pattern with a windspeed ratio of less than or equal to 0.25, which will provide a reasonable amount of outbound time before turning back to re-intercept the inbound course.

(3) The exact solution of the holding pattern shows when flying the standard Type-1 holding pattern, the recommendation in the AIM for the OWCA, i.e. using an OWCA=3*IWCA is valid under limited conditions. These conditions were determined to be

a. $70 \le \alpha \le 95$ for $\bar{V}_W$ less than 0.3

b. $0 \le \alpha \le 180$ degrees, for $\bar{V}_W \le 0.05$

Using the exact solution of the holding pattern problem, we developed a "Smart-Convergence" algorithm, which drives to the correct holding pattern solution in a minimum number of circuits. The algorithm introduces the concept of the "Coupling-Effect", which shows that any changes in the outbound time to converge to the holding pattern solution will cause changes in both inbound time and undershoots/overshoots to the inbound course. In addition, any changes in the OWCA will cause changes in both inbound time and undershoots/overshoots to the inbound course. The "Coupling-Effect" is the root cause of why IFR Pilots take additional circuits in the holding pattern to converge to the correct holding pattern when the windspeed ratio is greater than about 0.1. We have shown that just the use of a single chart for the outbound time as a function of the $\bar{V}_W$ and α can reduce the number of circuits required to converge to the "Holding Pattern" solution by 40% at large windspeed ratios (i.e. 0.3). In addition, eyeballing the outbound time and the M-Factor from Figures 18(a) and (b) will allow IFR Pilots to converge to the correct holding pattern within two circuits. Clearly, the best option to reduce the IFR Pilot workload while attempting to converge to the correct holding pattern is to implement the equations for the outbound time and outbound heading directly into the GPS software. This would provide the Pilot with the needed information to fly the holding pattern without spending time using the "Trial and Error" method.

Finally, having the exact solution to the holding pattern as a function of $\bar{V}_W$ and α during training of IFR Pilots can eliminate many of the incorrect rules-of-thumb used by CFI-I's and the FAA, which relate to timing and wind correction in the holding pattern.

8.0 References

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